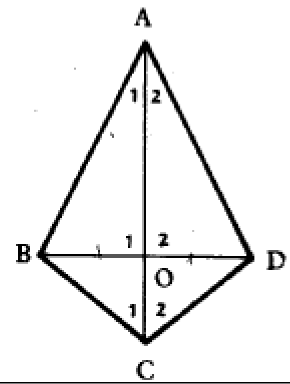
1. Prove the kite theorem
Given: $AB = AD$, $BC = DC$
Prove:
   a. $\measuredangle A_1 = \measuredangle A_2$, $\measuredangle C_1 = \measuredangle C_2$
   b. $BO = OD$
   c. $\measuredangle O_1 = \measuredangle O_2 = 90°$



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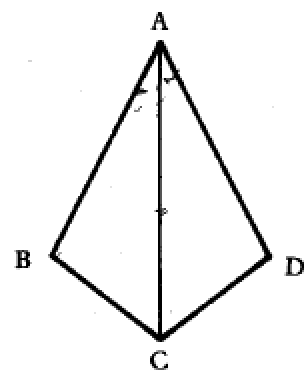
2. Given: AC bisects $\measuredangle A$ and $\measuredangle C$
Prove: ABCD is a kite



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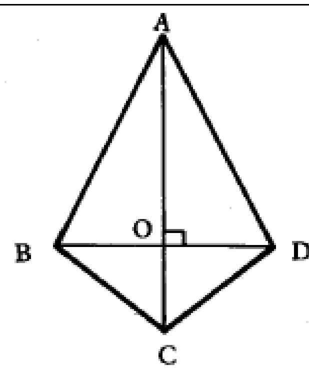
3. Given: $AC \perp BD$, $BO = OD$
Prove: ABCD is a kite



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4. In quadrilateral ABCD, the diagonal AC bisects $\measuredangle A$, and is perpendicular to diagonal BC. Prove that ABCD is a kite.

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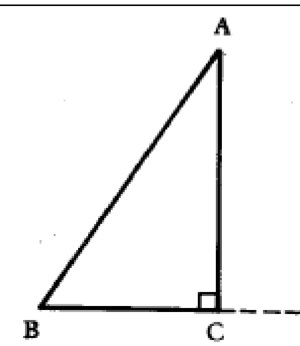
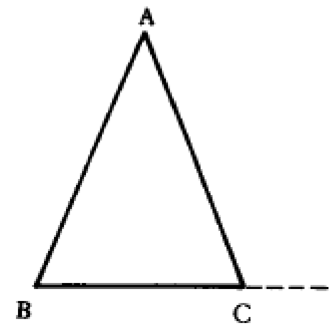
5. a. Prove: If a triangle has a right angle, then it is the largest angle of the triangle.
   Given: $\measuredangle C = 90°$
   Prove: $\measuredangle A < \measuredangle C$, $\measuredangle B < \measuredangle C$
   Guidance: Start with the external angle at vertex C.
   b. Prove section a's claim for any obtuse angle.



6. a. Prove: the base angles of an isosceles triangle are acute angles.
Given: $AB = AC$
Prove: $\measuredangle B < 90°$, $\measuredangle C < 90°$
Guidance: Start with the external angle at vertex C.
b. Prove: the sum of two angles in a triangle is smaller than 180°



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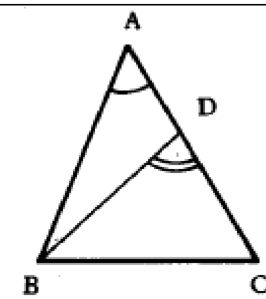
7. D is an arbitrary point on edge AC, a part of triangle ABC.
Prove: $\measuredangle A < \measuredangle BDC$



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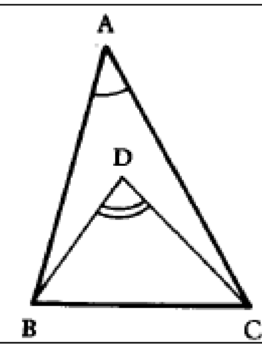
8. D is a point inside triangle ABC.
Prove: $\measuredangle A < \measuredangle BDC$



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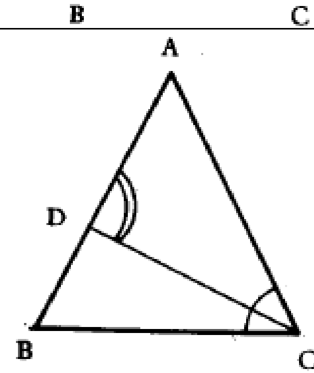
9. Given: $AB = AC$
D is an arbitrary point on edge AB
Prove: $\measuredangle ACB < \measuredangle ADC$



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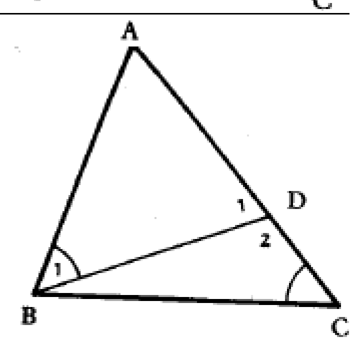
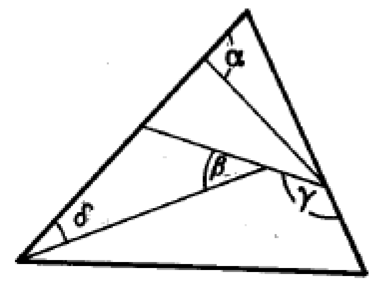
10. Given: $\measuredangle B_1 = \measuredangle C$
Prove:
a. $\measuredangle B_1 < \measuredangle D_1$
b. $\measuredangle C < \measuredangle D_2$



11. Prove:
    a. $\beta < \alpha$
    b. $\delta < \gamma$
    c. $\beta < \gamma$



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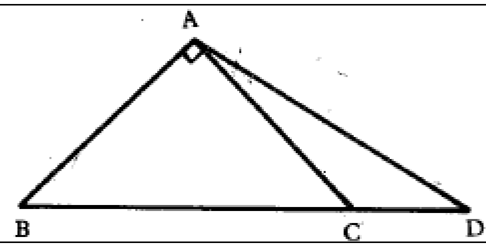
12. Given: $\measuredangle BAC = 90°$
    Prove:
    a. $AD < DB$
    b. $AC < AD$



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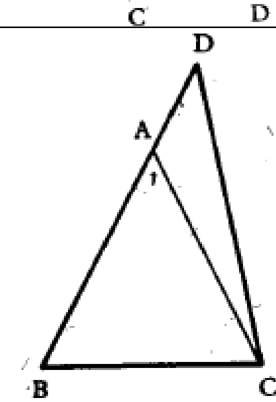
13. Given: $AB = AC$, $\measuredangle A_1 < \measuredangle B$
    Prove:
    a. $DC < DB$
    b. $BC < DC$



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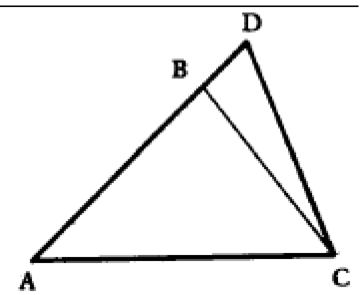
14. Given: $BC < AB$
    Prove:
    a. $DC < AD$
    b. Is the opposite claim also correct? Meaning: does $BC < AB$ emanate from $DC < AD$? Explain.



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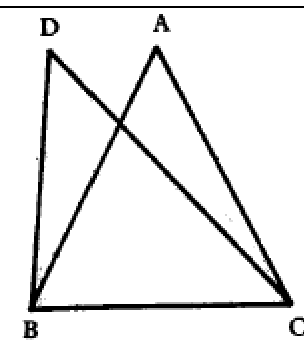
15. Given: Triangle ABC is isosceles.
    D is a point external to the triangle such that segment DC intersects with segment AB.
    Prove: $DB < DC$



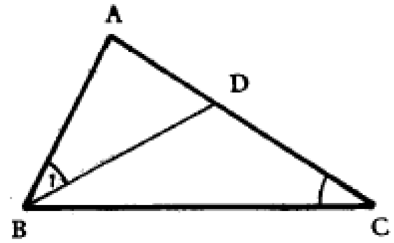
16. Given: $\measuredangle B_1 = \measuredangle C$

Prove:

a. $AB < AC$

b. $AD < AB$

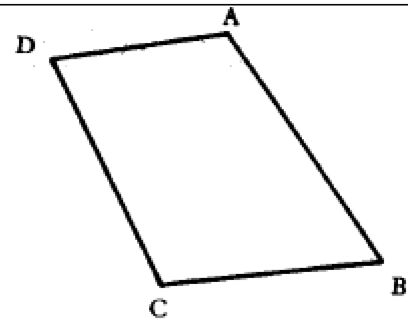
c. $BD < BC$



17. In quadrilateral ABCD it is given that:

$BC < AB$, $\measuredangle C < \measuredangle A$

Prove: $AD < DC$

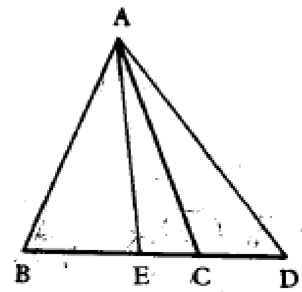


18. Given: Triangle ABC is isosceles ($AB = AC$). E is a point between B and C. D lies on the continuation of BC.

Prove:

a. $AE < AC$

b. $AC < AD$

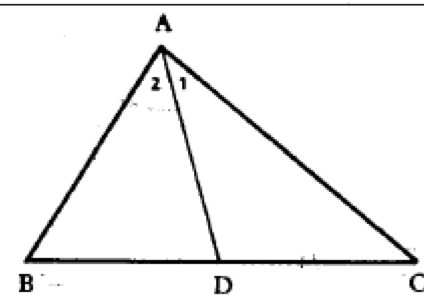


19. Given: AD is a median to edge BC.

$\measuredangle A_1 < \measuredangle A_2$

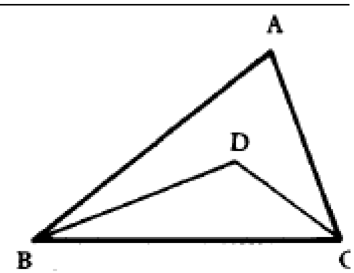
Prove: $AB < AC$

Guidance: Lengthen AD to twice is length, from D.



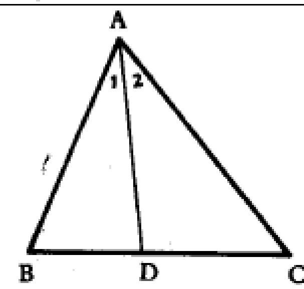
20. Given: BD and DC bisect angles $\measuredangle B$ and $\measuredangle C$, respectively. $AC < AB$

Prove: $DC < BD$

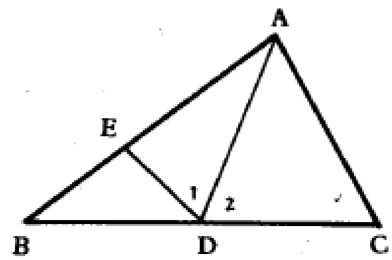


21. Given: AD bisects angle $\measuredangle A$ ($\measuredangle A_1 = \measuredangle A_2$).

Prove: $DC < AC$, $BD < AB$



22. Given: Segment AD bisects angle $\measuredangle EDC$ ($\measuredangle D_1 = \measuredangle D_2$).
Prove: $ED < BD$



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23. Prove the following theorem:
Theorem: Two right triangles, with hypotenuses of equal length, and one additional side of equal length, are congruent.

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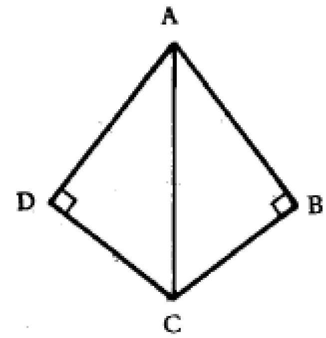
24. Given: $\measuredangle B = \measuredangle D = 90°$, $AB = AD$
Prove:
   a. $BC = DC$
   b. AC bisects angles $\measuredangle C$ and $\measuredangle A$



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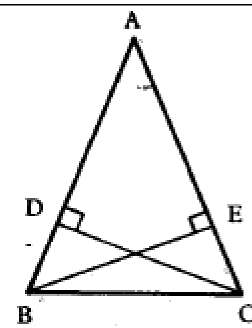
25. Prove: If a triangle has two altitudes of equal length, then the triangle is isosceles.
Given: $BE = DC$, $AB \perp DC$, $AC \perp BE$
Prove: $AB = AC$



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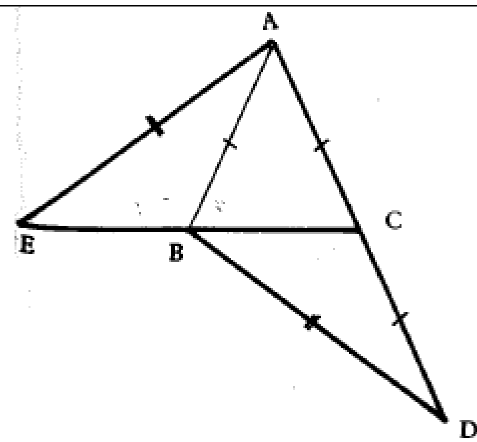
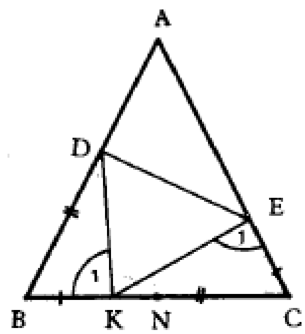
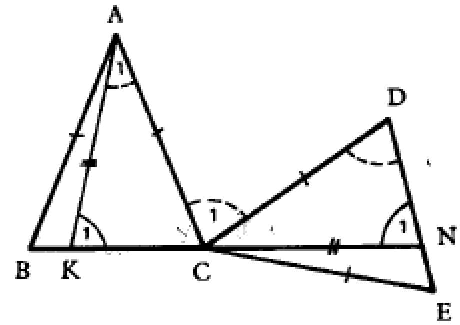
26. Given: $AB = AC = CD$, $AE = BD$
Prove: $EB = BC$



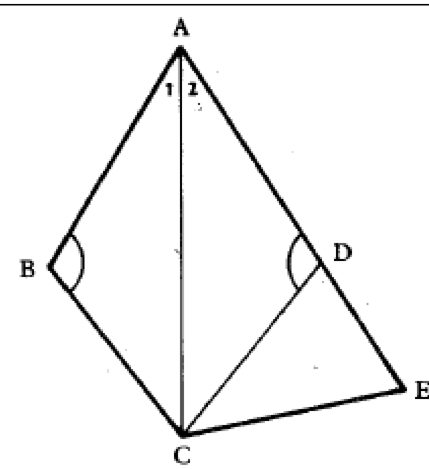
27. Point N is the middle of edge DC. Point K is between N and B.
Given: $KC = BD$, $BK = EC$, $\measuredangle K_1 = \measuredangle E_1$
Prove: Triangles ABC and DKE are isosceles.



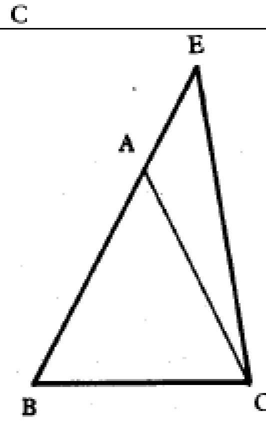
28. Given: $AB = AC = CD = CE$, $\measuredangle K_1 = \measuredangle N_1$, $AK = CN$
Prove:
   a. $KC = DN$
   b. $\measuredangle A_1 + \measuredangle C_1 + \measuredangle D = 180°$



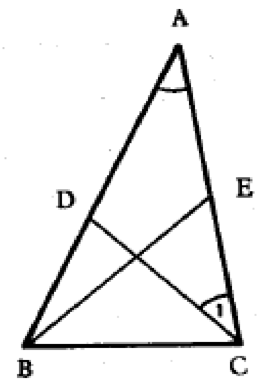
29. Given: $\measuredangle ADC = \measuredangle B$, $BC = CD = CE$
Prove:
   a. $\measuredangle A_1 = \measuredangle A_2$
   b. Find in the drawing a triangle that has two edges and an angle that are equal to two edges and an angle of triangle ABC, respectively, but is not congruent to triangle ABC.
   c. $\measuredangle B + \measuredangle E = 180°$



30. Given: Triangle ABC is isosceles ($AB = AC$).
Prove: $AC + AE < BC + CE$



31. Given: BE is the median to edge AC in triangle ABC.
$\measuredangle A = \measuredangle C_1$
Prove: $BD + DC < BE + EC$



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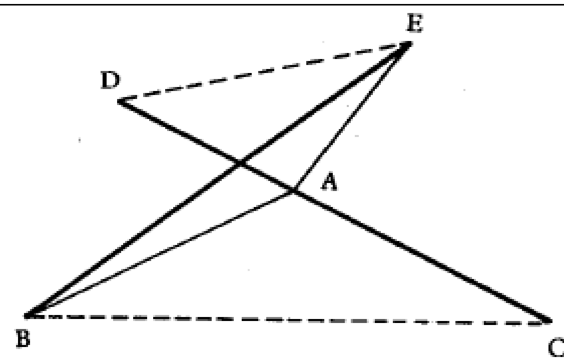
32. Given: A is a point on edge DC.
$AB = AC$. $AD = AE$
Prove:
a. $BE < DC$
b. $\dfrac{BC + DE}{2} < DC$



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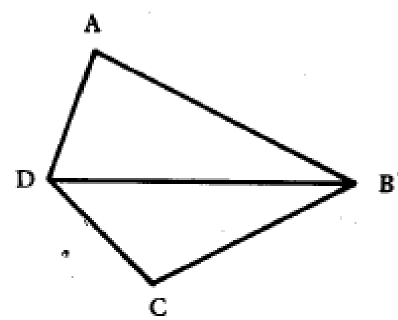
33. Given: BD bisects angle $\measuredangle B$. $CB < AB$
Prove: $DC + BC < AD + AB$
Guidance: Copy BC onto AB from point B.



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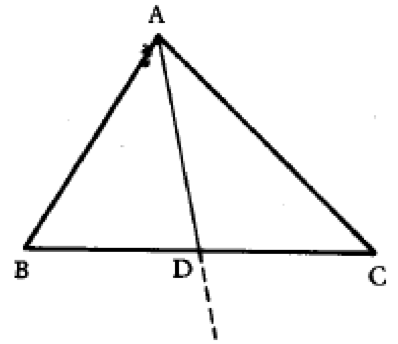
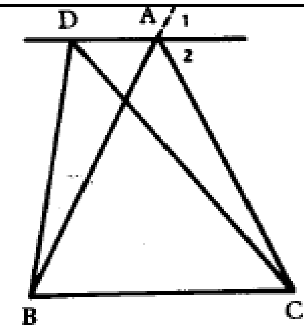
34. a. Prove: Each edge of a triangle is smaller than half of the perimeter of the triangle.
Guidance: Prove using an algebraic calculation, and the theorem on the sum of two edges of a triangle.
b. A point inside a triangle is given.
Prove: the sum of the distances from the point to the vertices is smaller than the perimeter of the triangle, and is larger than half of the perimeter.
c. Prove: the sum of the lengths of the diagonals of a quadrilateral is smaller than the perimeter of the quadrilateral.

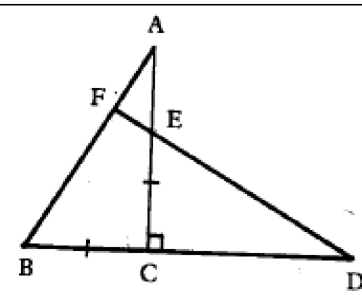
35. Prove the theorem: The length of the median to an edge of a triangle is smaller than half of the sum of the lengths of the other two edges.
Given: $BD = DC$
Prove: $AD < \frac{AB + AC}{2}$
Guidance: Lengthen the median to double its length, from D.



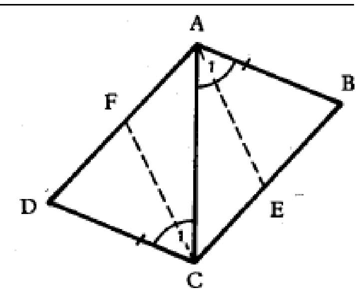
36. Triangle ABC is isosceles ($AB = AC$). The continuation of segment DA bisects the external angle that is next to A ($\measuredangle A_1 = \measuredangle A_2$).
Prove: $AB < \frac{DB + DC}{2}$
Guidance: On the continuation of AB, mark segment AE, equal to segment AB in length, and connect E with D.



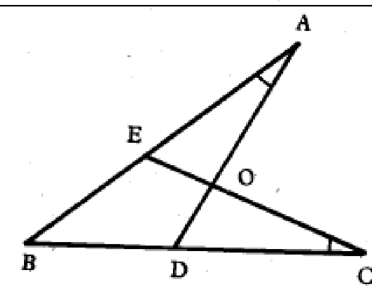
37. Given: $\measuredangle ACD = 90°$, $CE = BC$, $AC = CD$
Prove:
a. $\triangle ABC \cong \triangle ECD$
b. $\measuredangle AEF = \measuredangle B$



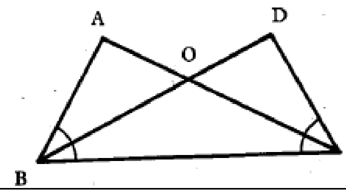
38. Given: $\measuredangle A_1 = \measuredangle C_1$, $AB = DC$
a. Prove: $AD = BC$
b. Given: E is the middle of BC, F is the middle of AD. Prove: $FC = AE$



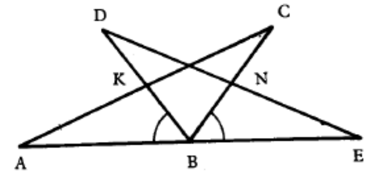
39. Given: $AB = BC$, $\measuredangle A = \measuredangle C$
Prove:
a. $AE = DC$
b. $OC = OA$



40. Given: BD bisects $\measuredangle ABC$, AC bisects $\measuredangle BCD$, $\measuredangle BCD = \measuredangle ABC$
Prove:
   a. $AC = BD$
   b. $OA = OD$

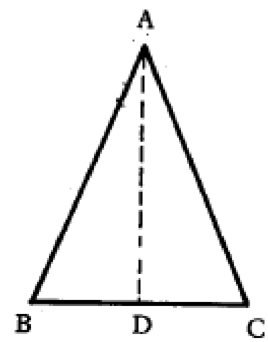


41. Given: $BC = BD$, B is the middle of AE, $\measuredangle CBE = \measuredangle ABD$
Prove:
   a. $AC = DE$
   b. $AK = NE$

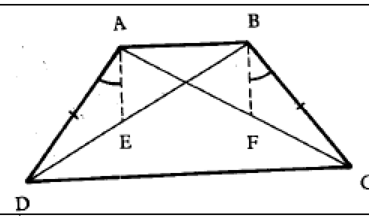


42. Prove the theorem: the base angles of a triangle are equal if and only if the triangles is isosceles.

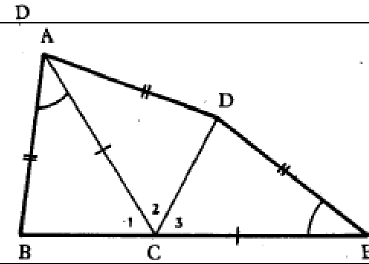
Given: $AB = AC$ Prove: $\measuredangle B = \measuredangle C$ Guidance: Draw AD, bisector of $\measuredangle A$	Given: $\measuredangle B = \measuredangle C$ Prove: $AB = BC$



43. Given: $AD = BC$, $AC = BD$, $\measuredangle DAE = \measuredangle CBF$
Prove:
   a. $\measuredangle ADE = \measuredangle BCF$
   b. $AE = BF$



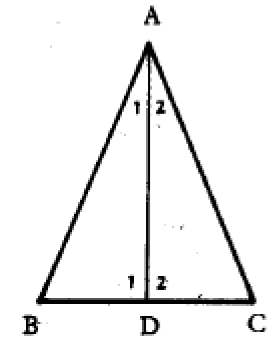
44. Given: $AC = CE$, $AB = AD = DE$, $\measuredangle BAC = \measuredangle E$
Prove:
   a. $\measuredangle C_1 = \measuredangle C_3$
   b. $\measuredangle C_2 = 60°$



45. Prove the theorem: In an isosceles triangle, the bisector to the head angle is also a median to the base and also an altitude to the base.
Guidance: Draw an appropriate sketch and list what is given and what needs to be proven.

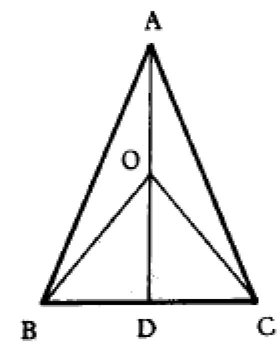
46. Prove: in an isosceles triangle, the median to the base also bisects the head angle and is also an altitude to the base.

47. Prove the theorem: If in a triangle an angle bisector is also an altitude, then the triangle is isosceles.
Given: $\measuredangle A_1 = \measuredangle A_2$, $\measuredangle D_1 = 90°$
Prove: $AB = AC$

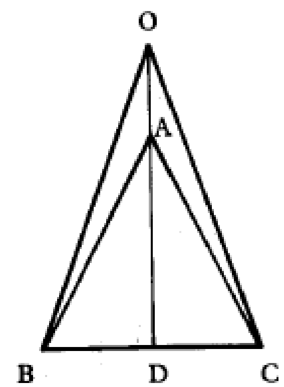


48. Prove: If in a triangle a median is also an altitude, then the triangle is isosceles.

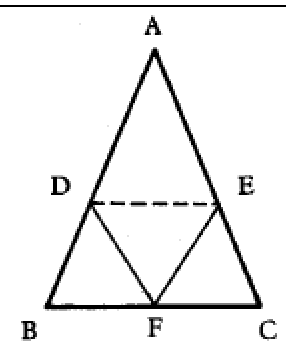
49. Given: $\triangle ABC$ is isosceles ($AB = AC$), AD bisects $\measuredangle A$, O is a point on the angle bisector.
Prove: $BO = OC$



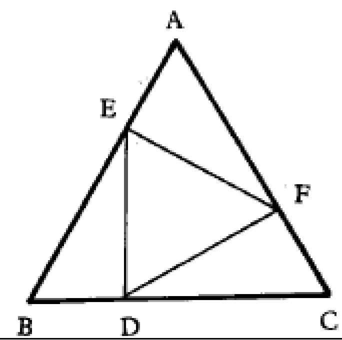
50. Given: $\triangle ABC$ is isosceles ($AB = AC$), AD is a median to base BC , O is on the continuation of AD.
Prove: $\triangle BOC$ is isosceles



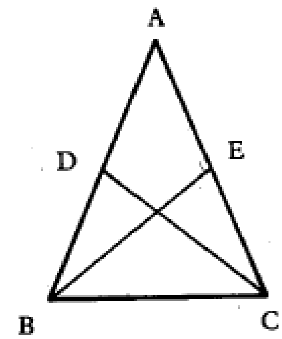
51. Given: $\triangle ABC$ is isosceles ($AB = AC$), $AD = AE$, F is the middle of BC.
Prove: $\triangle DEF$ is isosceles



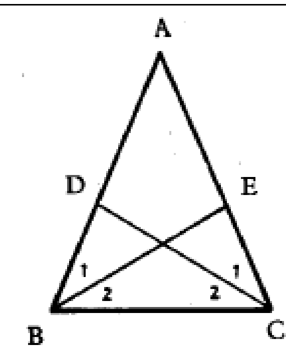
52. Given: $\triangle ABC$ is equilateral.
   a. Prove: $\measuredangle A = \measuredangle B = \measuredangle C$
   b. Given: $BD = AE = CF$
      Prove: $\triangle EFD$ is equilateral.



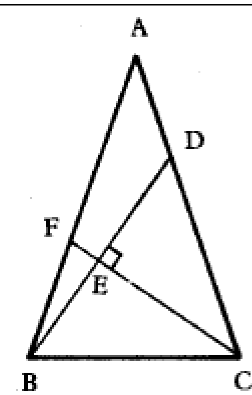
53. Prove: In an isosceles triangle the medians to the sides are equal in length.
Given: $AB = AC$, $AD = DB$, $AE = EC$
Prove: $DE = BE$



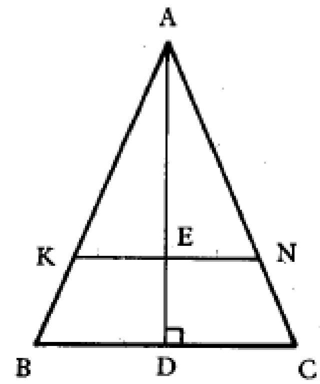
54. Prove: In an isosceles triangle the bisectors of the base angles are equal in length.
Given: $AB = AC$, $\measuredangle B_1 = \measuredangle B_2$, $\measuredangle C_1 = \measuredangle C_2$
Prove: $DC = BE$



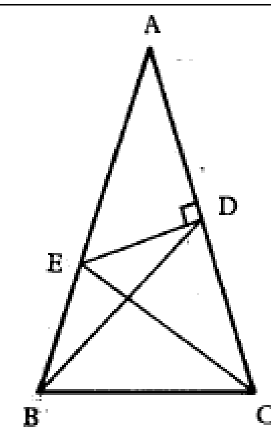
55. In isosceles triangle ABC ($AB = AC$) the segment FC is perpendicular to BD and divides it evenly in two ($BE = ED$).
Prove: $AD + BC = AB$



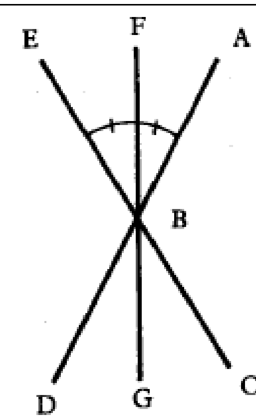
56. Given: $AK = AN$, E is the center of KN, $AD \perp BC$
Prove: $KB = NC$



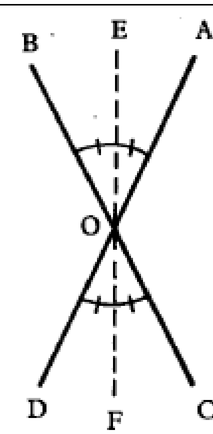
57. Given: $\triangle ABC$ is isosceles ($AB = AC$), BD is a median to AC, $ED \perp AC$
Prove: $DC = \dfrac{EC + EB}{2}$



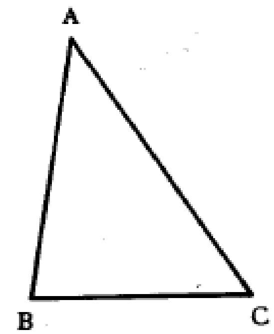
58. Given: segment FG bisects $\measuredangle EBA$
Prove: FG also bisects $\measuredangle DBC$



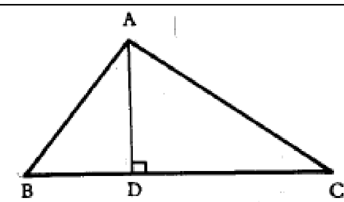
59. Given: EO bisects $\measuredangle BOA$, FO bisects $\measuredangle DOC$
Prove: points E, O and F line on the same line.
Guidance: Prove: $\measuredangle EOF = 180°$



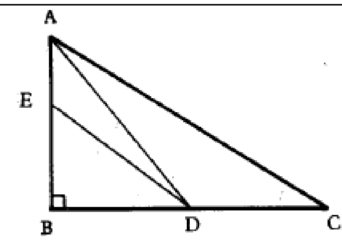
60. Prove the theorem: If one angle in a triangle is larger than another angle in the triangle, then the side that is opposite the larger angle is longer than the side that is opposite the smaller angle.
Given: $\measuredangle C < \measuredangle B$
Prove: $AB < AC$
Guidance: Assume (incorrectly): $AC = AB$ or $AC < AB$ and receive a contradiction to the opposite theorem.



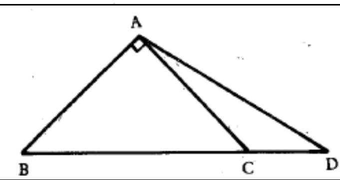
61. Given: AD is an altitude to BC
   a. Prove: $DC < AC$, $BD < AB$
   b. Prove: the altitude to one side is shorter then half of the sum of the other two sides
      Guidance: Prove: $AD < \frac{AB + AC}{2}$



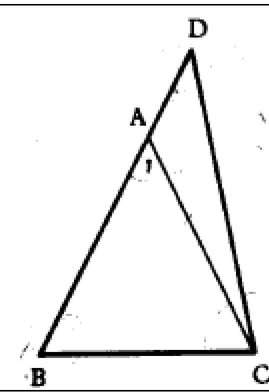
62. Given: $\triangle ABC$ is a right triangle ($\measuredangle = 90°$), D and E are on BC and AB, respectively.
Prove: $ED < AD < AC$



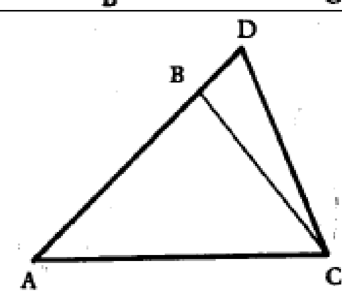
63. Given: $\measuredangle BAC = 90°$
Prove:
   a. $AD < DB$
   b. $AC < AD$



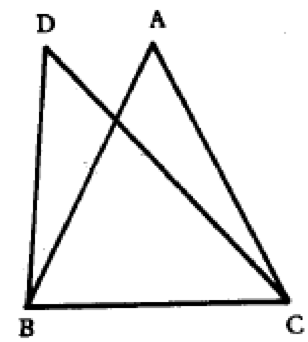
64. Given: $AB = AC$, $\measuredangle A_1 < \measuredangle B$
Prove:
   a. $DC < DB$
   b. $BC < DC$



65. Given: $BC < AB$
Prove:
   a. $DC < AB$
   b. Is the opposite claim also correct, namely: $DC < AD \Rightarrow BC < AB$? Explain.



66. Given: $\triangle ABC$ is isosceles ($AB = AC$). D is a point external to the triangle such that segment DC intersects with segment AB.
Prove: $DB < DC$



67. Given: $\measuredangle B_1 = \measuredangle C$
Prove:
a. $AB < AC$
b. $AD < AB$
c. $BD < BC$