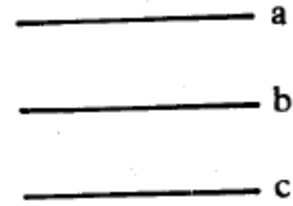


1. Given: $a \parallel b$, $b \parallel c$

Prove: $a \parallel c$

Guidance: Draw a line which intersects with all three lines.

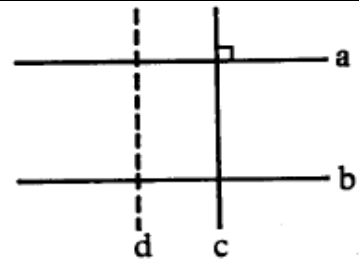


2. Given: $a \parallel b$, $c \perp a$

a. Prove: $c \perp b$

b. Given: $d \perp b$

Prove: $d \parallel c$

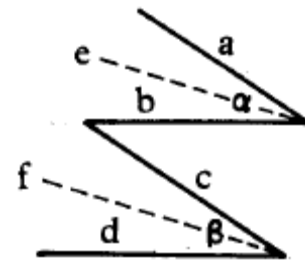


3. Given: $a \parallel c$, $b \parallel d$

a. Prove: $\alpha = \beta$

b. Given: e and f bisect angles α and β respectively.

Prove: $e \parallel f$



4. Prove the theorem: Angles, whose sides are parallel, respectively, are equal to each other, or their sum is 180°

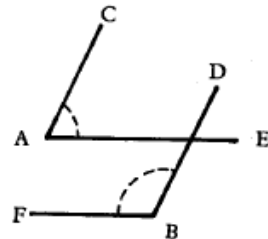
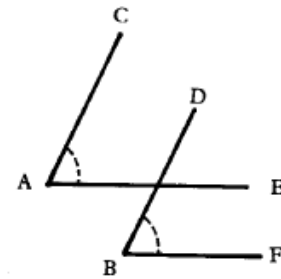
Guidance:

First case: Given: $AC \parallel BD$, $AE \parallel BF$; Prove:

$\angle A = \angle B$

Second Case: Given: $AC \parallel BD$, $AE \parallel BF$; Prove:

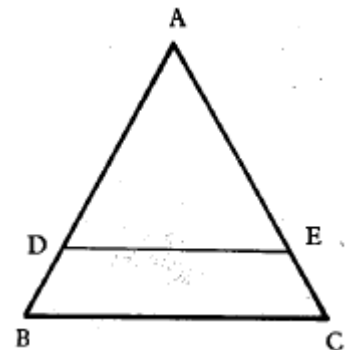
$\angle A + \angle B = 180^\circ$



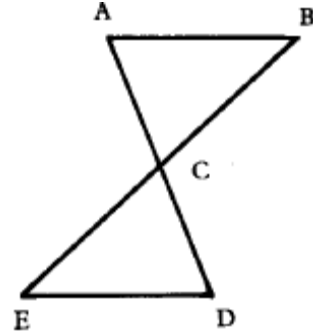
5. $\triangle ABC$ is isosceles ($AB = AC$)

Given: $DE \parallel BC$

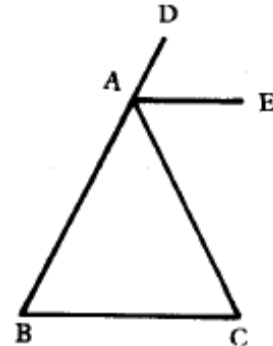
Prove: $\triangle ADE$ is isosceles



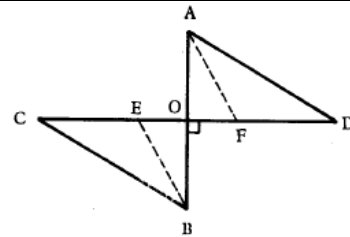
6. Given: $AB \parallel ED$, $AC = CD$
 Prove: $EC = CB$



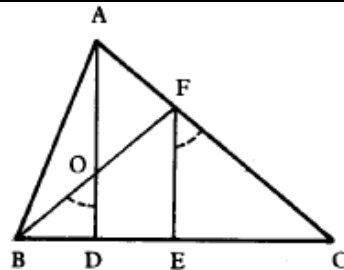
7. Given: The segment AE is parallel to BC and bisects $\angle DAC$
 Prove: $\triangle ABC$ is isosceles



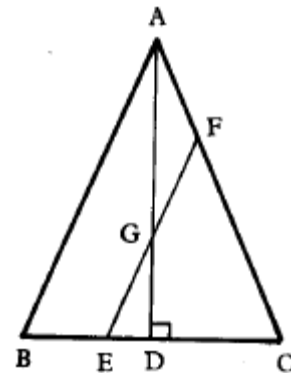
8. Given: $AB \perp CD$, $AD = CB$, $AO = OB$
 a. Prove: $AD \parallel BC$
 b. Given: AF bisects $\angle A$ and $BE \parallel AF$
 Prove: BE bisects $\angle B$



9. Given: $EF \parallel AD$, $FB = FC$, $BE = EC$
 Prove:
 a. AD is an altitude to side BC in triangle ABC
 b. $\angle BOD = \angle EFC$



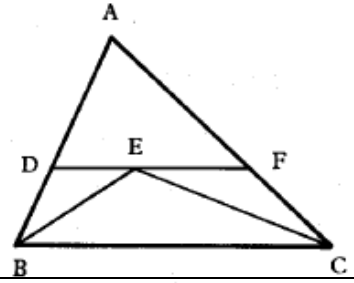
10. Given: $\triangle ABC$ is isosceles ($AB = AC$), AD is an altitude to BC, EF is parallel to AB.
 Prove:
 a. $\triangle AFG$ is isosceles.
 b. $AB = AF + FE$



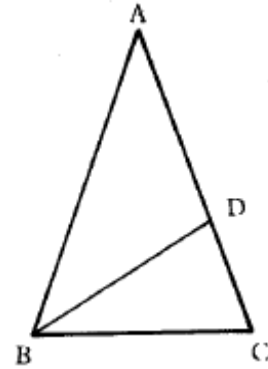
11. Given: BE and EC bisect $\angle B$ and $\angle C$, respectively, and meet at point E. Segment DF passes through E, and is parallel to BC.

Prove:

- $BD = DE$
- $DF = BD + FC$

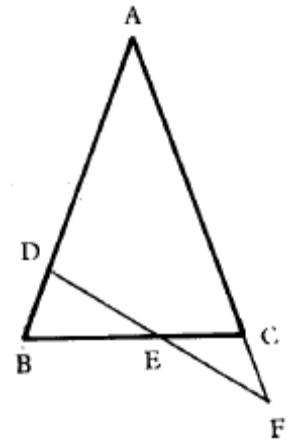


12. Given: $AD = DB = BC$, $AB = AC$
Prove that BD bisects $\angle B$ and calculate $\angle A$

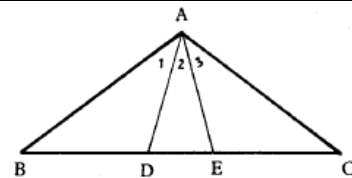


13. $\triangle ABC$ is isosceles ($AB = AC$). From point F on side AB, there is an altitude to AB, which intersects the base BC at point E, and intersects the continuation of side AC at point D. Given: $EC = CD$. Calculate $\angle A$.

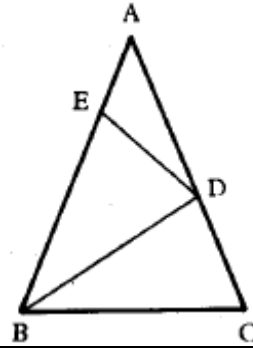
14. Given: $AB = AC$, $AD = DF$, $EC = FC$
- Prove: $\angle ADF = 3\angle F$
 - Calculate $\angle B$



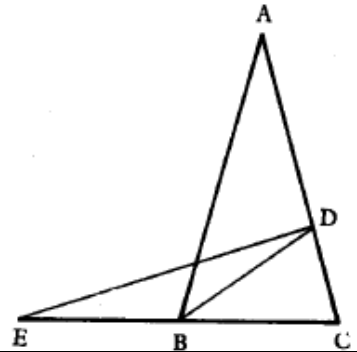
15. Given: $BD = AD = AE = EC$
- Prove: $\angle A_1 = \angle A_3$
 - Given: $\angle A_1 = \angle A_2$
Calculate $\angle A$



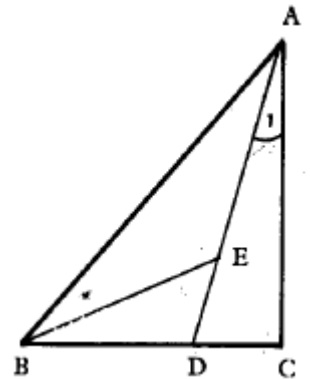
16. Given: $AB = AC$, $EB = BC$, $ED = DC$, $\angle A = 40^\circ$
 Calculate $\angle ADE$ and $\angle DBC$



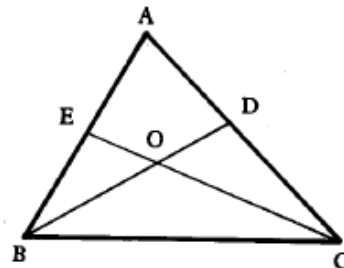
17. Given: $AB = AC$, $EB = BD$, $\angle E = \angle A$, BD bisects $\angle ABC$
 Calculate $\angle A$



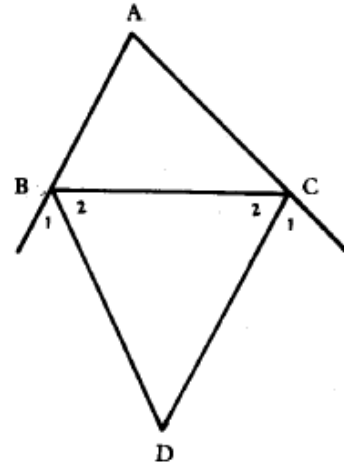
18. Given: $AE = BE$, BE bisects $\angle ABC$
 a. Prove: $\angle ADC = 3\angle ABE$
 b. Given: $\angle C = 90^\circ$, $\angle A_1 = 15^\circ$
 Calculate $\angle ABD$



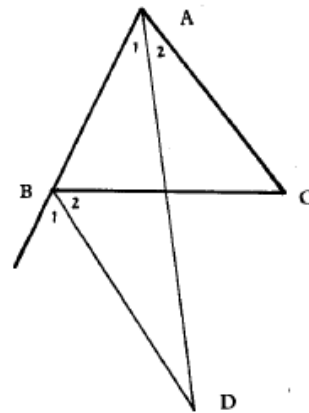
19. Given: BD and EC bisect $\angle B$ and $\angle C$ respectively,
 $\angle A = 70^\circ$
 Calculate $\angle BOC$
 Note: the triangle is not isosceles.



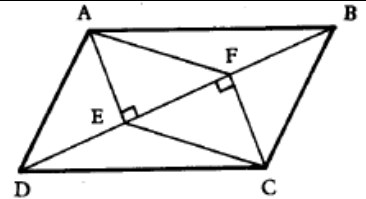
20. Given: $\angle A = 80^\circ$, DB and DC bisect the external angles to $\angle B$ and $\angle C$ ($\angle B_1 = \angle B_2$, $\angle C_1 = \angle C_2$)
Calculate $\angle D$.



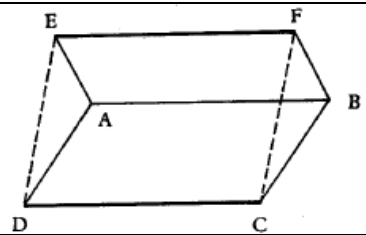
21. Given: $\angle C = 50^\circ$, $\angle A_1 = \angle A_2$, $\angle B_1 = \angle B_2$
a. Calculate $\angle D$
b. Prove: $\angle D = \frac{1}{2} \angle C$ without using the given that $\angle C = 50^\circ$.



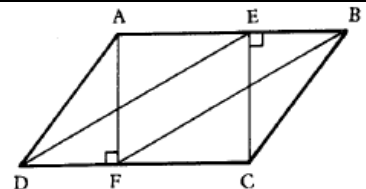
22. Given: Quadrilateral ABCD is a parallelogram,
 $AE \perp DB$, $CF \perp DB$
Prove: AFCE is a parallelogram.



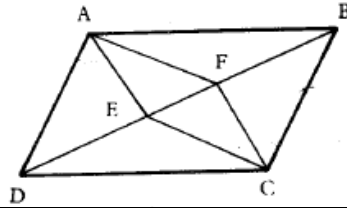
23. Given: Quadrilaterals ABCD and EFBA are parallelograms.
Prove: Quadrilateral EFCD is a parallelogram



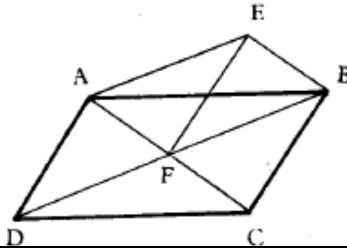
24. Given: ABCD is a parallelogram, $AF \perp DC$,
 $EC \perp AB$
Prove: EBFD is a parallelogram.



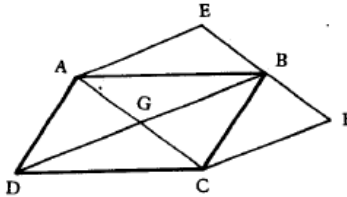
25. Given: Quadrilateral ABCD is a parallelogram, AE and FC bisect $\angle A$ and $\angle C$, respectively.
 Prove: AFCE is a parallelogram.



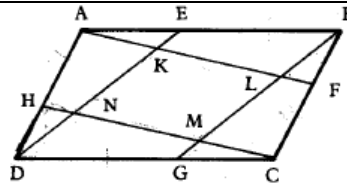
26. Given: Quadrilaterals ABCD and EBCF are parallelograms.
 Prove: EBFA is a parallelogram.



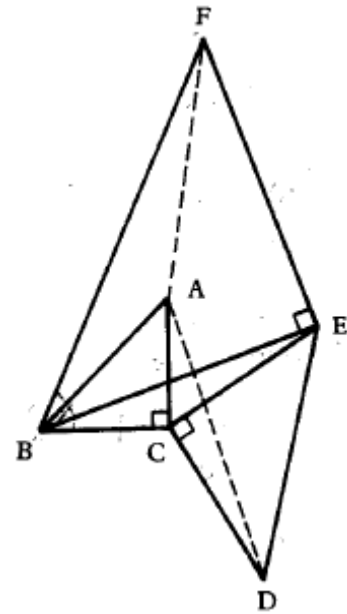
27. Given: Quadrilaterals ABCD and BFCG are parallelograms, $EB = BF$.
 Prove: AEFG is a parallelogram.



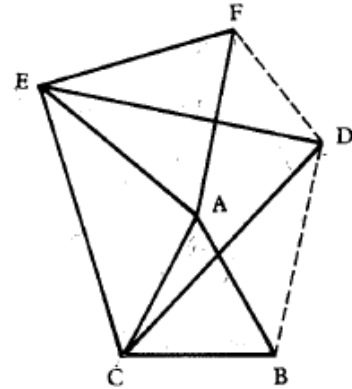
28. Given: Quadrilateral ABCD is a parallelogram,
 $EB = DG$, $AH = FC$
 Prove: KLMN is a parallelogram.



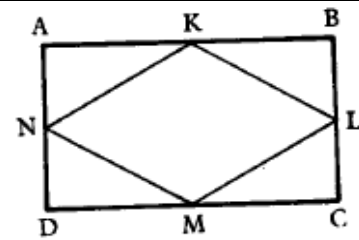
29. Given: $\triangle ABC$, $\triangle CDE$ and $\triangle BEF$ are isosceles and their head angles are right angles.
 Prove:
 a. $BE = AD$
 b. Quadrilateral AFED is a parallelogram



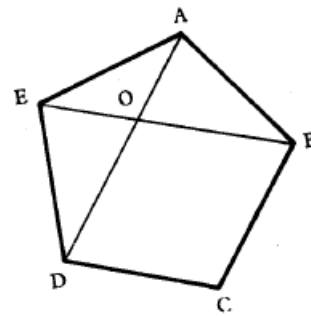
30. Given: $\triangle ABC$, $\triangle EDC$ and $\triangle EAF$ are equilateral.
 Prove:
 a. $BD = AE$
 b. Quadrilateral $FDBA$ is a parallelogram



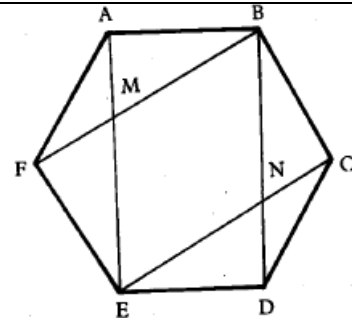
31. Given: The points K , L , M and N are the midpoints of the sides of rectangle $ABCD$.
 Prove: $KLMN$ is a rhombus.



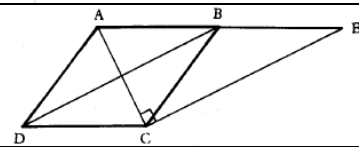
32. Given: pentagon $ABCDE$ is a regular pentagon.
 a. Calculate $\angle C$
 b. Prove: $OBCD$ is a rhombus



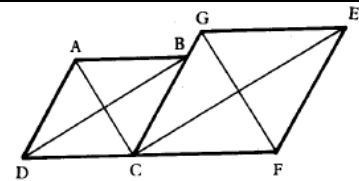
33. $ABCDEF$ is a regular hexagon.
 Prove:
 a. $ABDE$ is a rectangle
 b. $MBNE$ is a rhombus



34. Given: Quadrilateral $ABCD$ is a rhombus, $CE \perp AC$
 Prove: $DB = CE$



35. Given: Quadrilaterals $ABCD$ and $GEFC$ are rhombuses.
 Prove:
 a. $DB \parallel EC$
 b. $\angle ACE = 90^\circ$

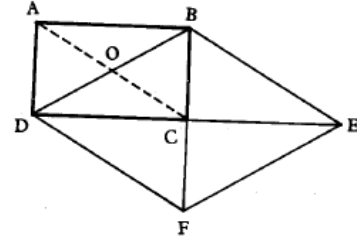


36. Given: Quadrilateral ABCD is a rectangle. Point C is the middle of both DE and BF.

Prove:

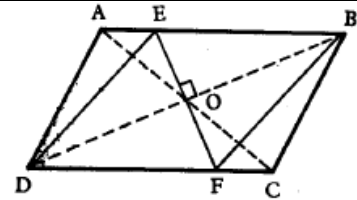
- a. BEFD is a rhombus.

b. $AO = \frac{BE}{2}$



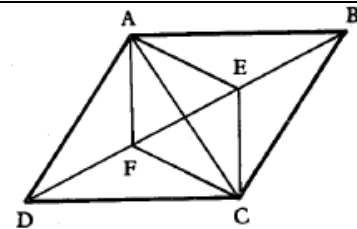
37. Given: Quadrilateral ABCD is a parallelogram, EF passes through point O and is perpendicular to BD.

Prove: EBFD is a rhombus.



38. Given: Quadrilateral ABCD is a rhombus, AE bisects $\angle CAB$, FC bisects $\angle ACD$.

Prove: AECF is a rhombus.

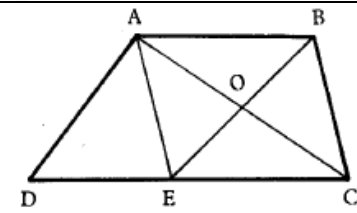


39. Given: Quadrilateral ABCD is a trapezoid, segment BE bisects diagonal AC ($AO = OC$).

Prove:

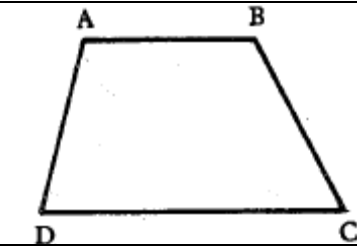
- a. Quadrilateral ABCE is a parallelogram

- b. It is impossible that one diagonal bisects another in a trapezoid.



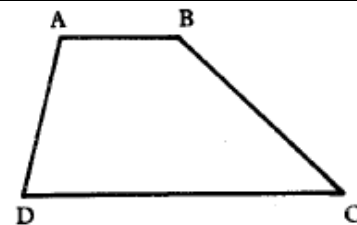
40. Given: Quadrilateral ABCD is a trapezoid, $AD < BC$

Prove: $\angle C < \angle D$



41. Given: Quadrilateral ABCD is a trapezoid

Prove: $DC - AB < AD + BC$, and express this statement in words

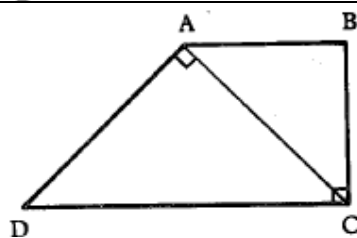


42. Given: Quadrilateral ABCD is a right trapezoid ($\angle C = 90^\circ$), $\angle DAC = 90^\circ$, $AB = BC$

Prove:

a. $AD = AC$

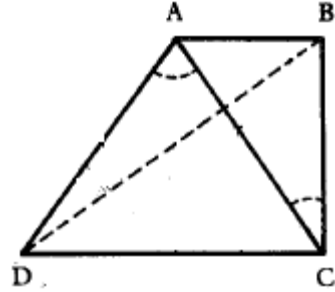
b. $AB = \frac{DC}{2}$



43. Given: Quadrilateral ABCD is a right trapezoid
 ($\angle B = 90^\circ$), $AD = AC$

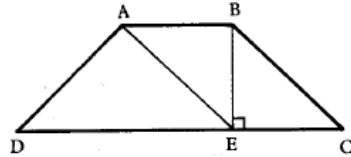
Prove:

- $\angle DAC = 2\angle ACB$
- Given: $BD \perp AC$, $\angle D_2 = 2\angle D_1$
 Calculate the angles of $\triangle ABC$.



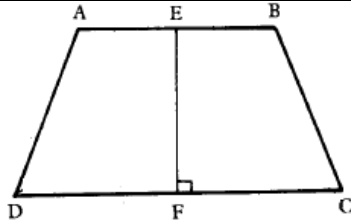
44. Given: Quadrilateral ABCD is an isosceles trapezoid,
 $BD = DC$, BD bisects $\angle D$

- Calculate the angles of the trapezoid.
- Given: $DB = a$, $BC = b$
 Express the perimeter of the trapezoid in terms of a and b .

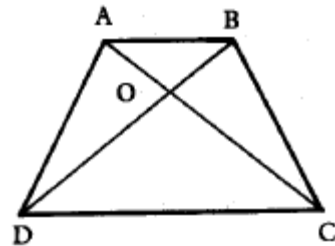


45. Given: Quadrilateral ABCD is a trapezoid, E and F
 are the centers of the bases AB and DC , respectively,
 EF is perpendicular to DC

Prove: Trapezoid ABCD is isosceles.



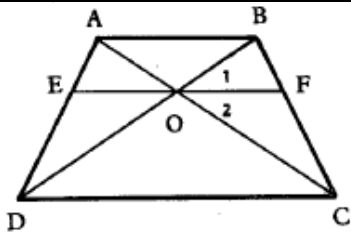
46. Given: Quadrilateral ABCD is an isosceles trapezoid.
 Prove: $DO = OC$, $AO = OB$



47. Given: Quadrilateral ABCD is an isosceles trapezoid.
 Segment EF passes through point O (the point of
 intersection of the two diagonals), $EF \parallel DC$

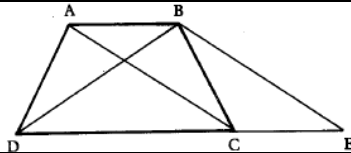
Prove:

- $\angle O_1 = \angle O_2$
- $EO = OF$



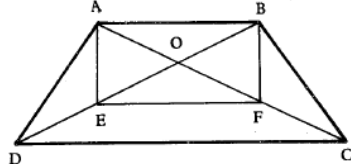
48. Given: Quadrilateral ABCD is an isosceles trapezoid,
 $CE = AB$ (CE is the continuation of DC)

Prove: $\triangle DBE$ is isosceles.

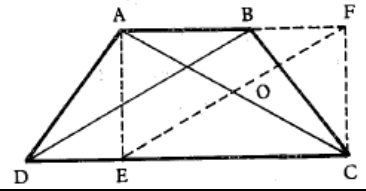


49. Given: Quadrilateral ABCD is an isosceles trapezoid,
 points E and F are the midpoints of segments DO and
 OC , respectively. $DO = 2OB$

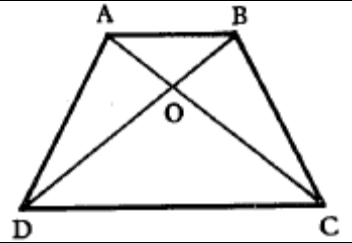
Prove: Quadrilateral ABFE is a rectangle.



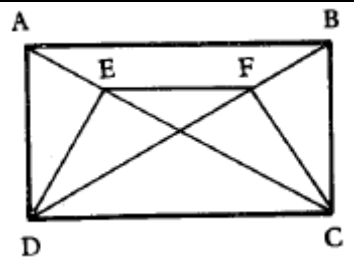
50. Given: Quadrilateral ABCD is an isosceles trapezoid, segment EF is parallel to diagonal BD and bisects diagonal AC ($AO = OC$). Segment BF is the continuation of AB.
 Prove: Quadrilateral AFCE is a rectangle.



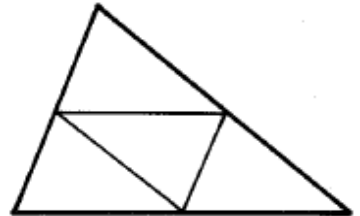
51. Given: Quadrilateral ABCD is a trapezoid,
 $DO = OC$
 Prove: the trapezoid is isosceles.



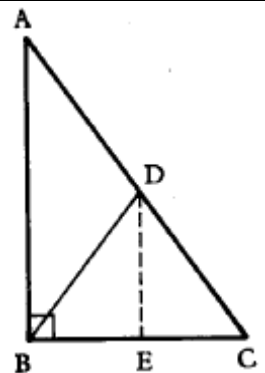
52. Given: Quadrilateral ABCD is a rectangle, $EF \parallel DC$
 Prove: EFCD is an isosceles trapezoid.



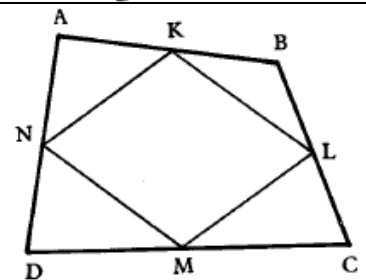
53. Prove the theorem: If we draw the three mid-segments in a triangle, we receive four congruent triangles.



54. Prove, using the properties of the mid-segment of a triangle, the theorem: the median to the hypotenuse of a right triangle is equal to half of the hypotenuse.
 Guidance: Draw mid-segment DE and prove that $\triangle BDC$ is isosceles.



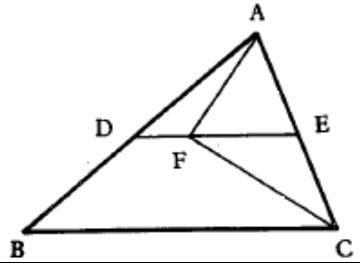
55. Given: ABCD is a quadrilateral, and points K, L, M and N are the midpoints of its edges.
 Prove: Quadrilateral KLMN is a parallelogram
 Guidance: Draw diagonal AC.



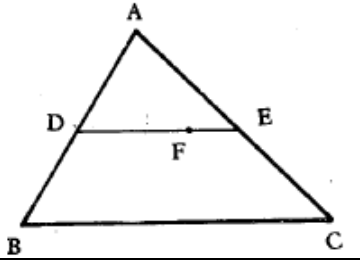
56. Prove: If we connect the midpoints, one after another, of the edges of a quadrilateral whose diagonals are equal, we receive a rhombus.

57. Prove: If we connect the midpoints, one after another, of the edges of a quadrilateral whose diagonals are perpendicular to one another, we receive a rectangle.

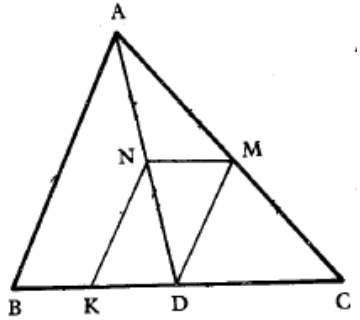
58. Given: DE is a mid-segment in $\triangle ABC$, CF bisects $\angle C$
 Prove: $\angle AFC = 90^\circ$



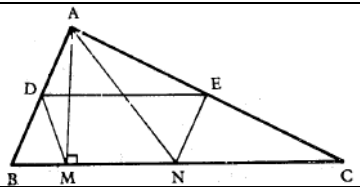
59. Given: DE is a mid-segment in $\triangle ABC$, $DB = DF$
 Prove: $FE < EC$



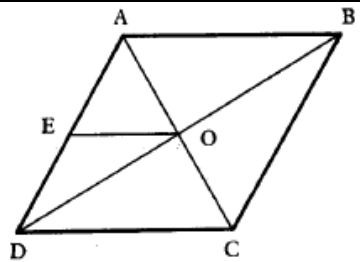
60. Given: segment AD is a median to side BC in $\triangle ABC$. K, M and N are the midpoints of segments BD, AC and AD, respectively.
 a. Prove: NMDK is a parallelogram
 b. What condition must $\triangle ABC$ fulfill so that NMDK will be a rectangle? Explain your answer.
 c. What condition must $\triangle ABC$ fulfill so that NMDK will be a rhombus? Explain your answer.



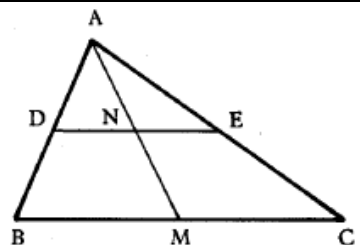
61. Given: DE is a mid-segment in $\triangle ABC$, AN is a median to BC, AM is an altitude to BC.
 Prove: DENM is an isosceles trapezoid.



62. Given: Quadrilateral ABCD is a rhombus and quadrilateral EOCB is an isosceles trapezoid.
 a. Prove: $AE = ED$
 b. Calculate the angles of the rhombus.
 c. Given: the perimeter of the rhombus is 24. Calculate the perimeter of the trapezoid.

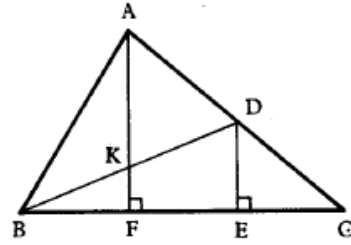


63. Given: AM is a median to edge BC in $\triangle ABC$, DE is a mid-segment in $\triangle ABC$.
 Prove: AN is a median to edge DE in $\triangle ADE$.



64. Given: segment AF is an altitude to edge BC in $\triangle ABC$, BD is a median to AC in the same triangle, and $ED \perp BC$

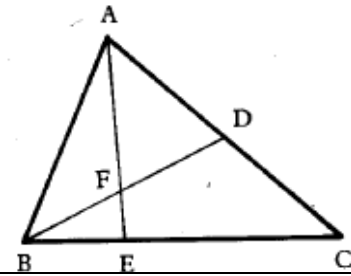
- Prove: $FE = EC$
- Given: $BF = FE$, $AF = 10\text{cm}$
Calculate the length of AK



65. Given: BD is a median to edge AC in $\triangle ABC$, segment AE bisects the median BD ($BF = FD$)

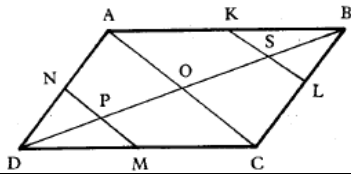
Prove: $BE = \frac{BC}{3}$

Guidance: Draw through D a line parallel to AE.



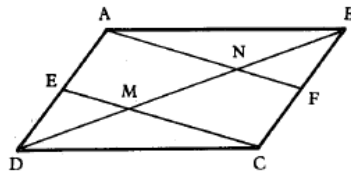
66. Given: Quadrilateral ABCD is a parallelogram. Points K, L, M and N are the midpoints of the edges of the parallelogram.

Prove: $DP = PO = OS = SB$



67. Given: E and F are the midpoints of edges AD and BC in parallelogram ABCD.

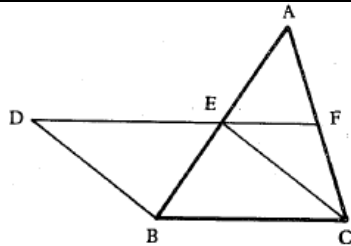
Prove: $DM = MN = BN$



68. Given: Quadrilateral DECB is a parallelogram,

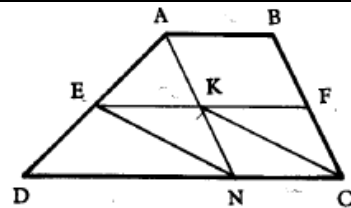
$EF = \frac{DE}{2}$ (EF is the continuation of DE)

Prove: EC is a median in $\triangle ABC$

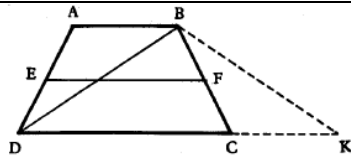


69. Given: EF is the mid-segment in trapezoid ABCD, the quadrilaterals ABCN and EKCN are parallelograms

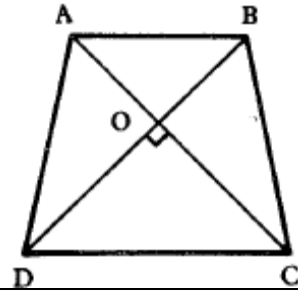
Prove: $DC = 3AB$



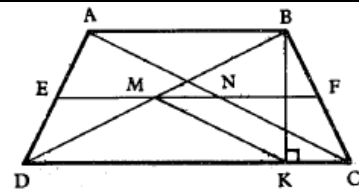
70. Prove the theorem: The diagonal of an isosceles trapezoid is longer than the mid-segment.
Guidance: Prove: $EF < BD$ (EF is the mid-segment).
Lengthen the base DC by the length of the base AB ($AB = CK$), and use $\triangle DBK$



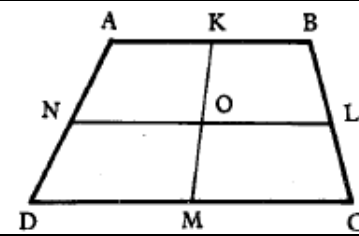
71. Given: In the isosceles trapezoid ABCD, the diagonals are perpendicular to one another ($AC \perp BD$).
 Prove: the height of the trapezoid is equal to the mid-segment.
 Guidance: draw the height through the point O.



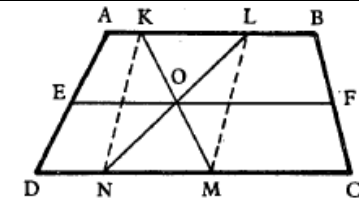
72. Given: EF is the mid-segment of the isosceles trapezoid ABCD, BK is an altitude.
 Prove: MNCK is a parallelogram.



73. Given: the points K, L, M and N are the midpoints of the edges of the trapezoid ABCD.
 Prove: $NO = OL$



74. Given: EF is the mid-segment in trapezoid ABCD. O is a point on EF, segments KM and NL pass through O
 Prove: KLMN is a parallelogram



75. Given: ABCD is an isosceles trapezoid, KMND is a parallelogram, and segment BN is an altitude in the trapezoid.
 Prove: KM is a mid-segment in trapezoid ABCD.

