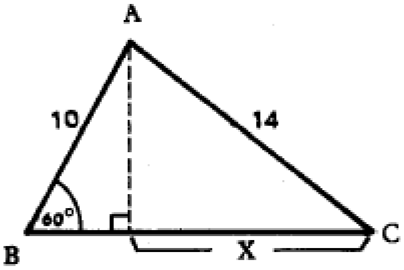
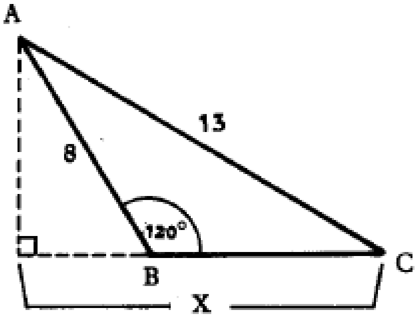
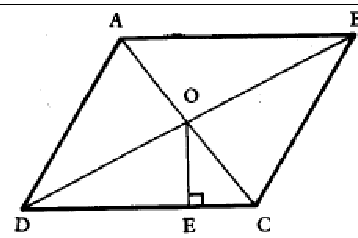
1. Find the length of BC in the following triangles. It will help to first find the length of the segment marked X.

a: 

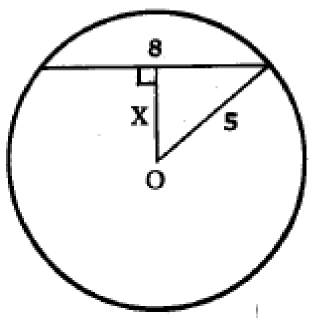
b: 

2. Given: the diagonals of parallelogram ABCD meet at point O. The altitude OE divides side DC into two segments: $DC = 14cm$, $EC = 6cm$. Also given: $AD = 17cm$.
Calculate the area of the parallelogram.

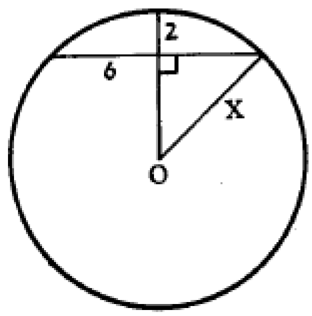


3. Calculate the area of an equilateral triangle whose side length is a.

4. Find X in the following drawings: (O is the center of the circle):



a:



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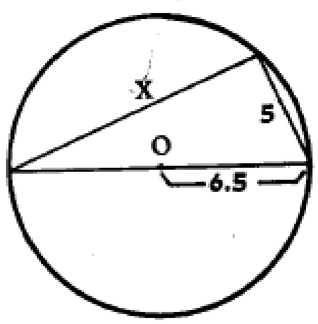

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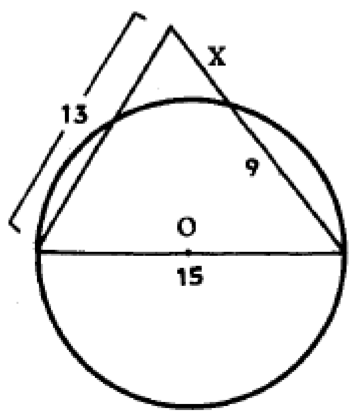
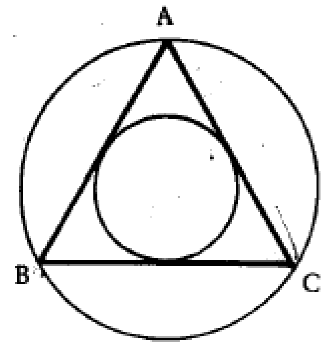
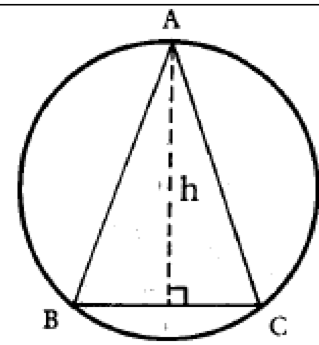


d:

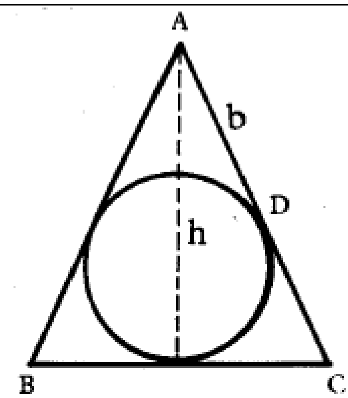
5. Given: $\triangle ABC$ is equilateral, and has edge length a.
   a. Express the radius of the circle that circumscribes the triangle in terms of a.
      Guidance: The center of the circle that circumscribes the triangle is also the point of intersection of the medians.
   b. Express the radius of the circle that is inscribed in the triangle, in terms of a.



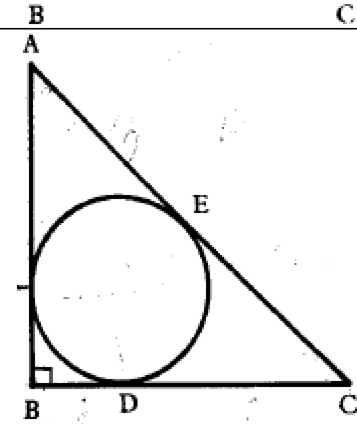
6. Given: $\triangle ABC$ is isosceles ($AB = AC$). The altitude to the base is h and the radius of the circle that circumscribes the triangle is R ($\measuredangle A < 90°$)
   Express the sides of the triangle in terms of h and R.



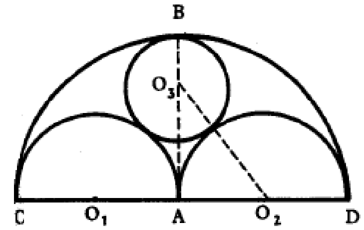
7. Given: $\triangle ABC$ is isosceles ($AB = AC$). D is the tangent point between the circle and side AC of the triangle. $AD = b$. The altitude to the base is h. Express, in terms of b and h, the radius of the inscribed circle and the base BC.



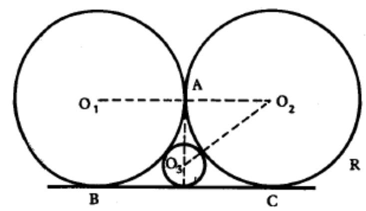
8. Given: $\triangle ABC$ is a right triangle ($\measuredangle B = 90°$), and in it a circle is inscribed. D and E are two tangent points between the circle and the triangle.
   a. Given: $AB = 15\text{cm}$, $BC = 8\text{cm}$.
      Calculate the radius of the inscribed circle.
   b. Note: the givens from section a no longer apply.
      Calculate the radius of the inscribed circle if:
      $AE = 3\text{cm}$, $EC = 2\text{cm}$.
   c. Note: the givens from sections a and b no longer apply.
      Given: $\triangle ABC$ is isosceles ($BA = BC \equiv a$).
      Express the length of segment DC and the radius of the inscribed circle in terms of a.



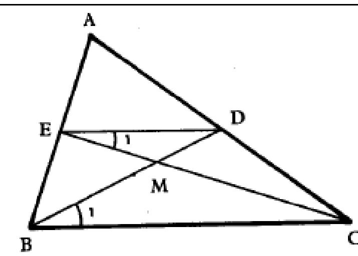
9. Two small semicircles, whose centers are O_1 and O_2, and who have equal radii, are inscribed in a large semicircle. The two small semicircles touch the large semicircle at points C and D, and touch each other at point A. An additional circle, whose center is at point O_3, touches both small semicircles and touches the large semicircle at point B.
   a. Prove: segment AB, which passes through O_3, is perpendicular to O_1O_2.
   b. Given: the radius of the large semicircle is equal to R.
   Express, in terms of R, the radii of the two small semicircles, and the radius of the circle whose center is O_3.



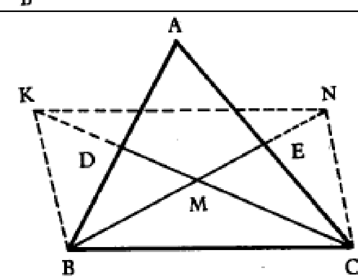
10. The radii of the circles whose centers are O_1 and O_2 are equal. The two circles are tangent to each other at point A and are tangent to line BC. A third circle, whose center is O_3, is tangent to the two circles and to line BC.
   a. Prove: AO_3 is perpendicular to O_1O_2.
   b. Given: the radius of each of the large circles is R.
   Express the radius of the small circle in terms of R.



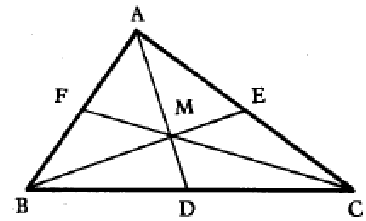
11. Given: EC and BD are medians in $\triangle ABC$ which meet at M.
   a. Prove: the perimeter of $\triangle EMD$ is equal to half of the perimeter of $\triangle BMC$
   b. Given: $BD < EC$
   Prove: $\measuredangle E_1 < \measuredangle B_1$



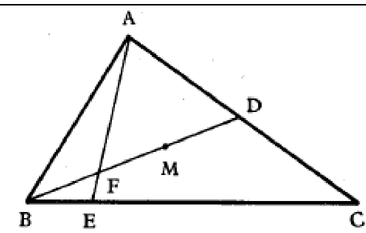
12. Given: BE and CD are medians in $\triangle ABC$ which meet at M. The medians were extended such that $ME = EN$ and $KD = DM$.
   a. Prove: the quadrilateral KNCB is a parallelogram.
   b. What condition does $\triangle ABC$ need to sustain such that the quadrilateral KNCB will be a rectangle? Explain.



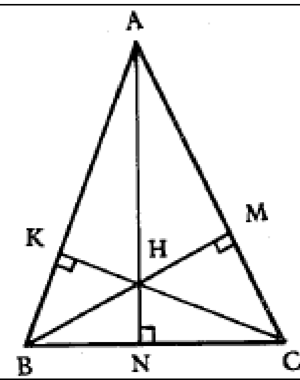
13. Given: AD, BE and CF are medians in $\triangle ABC$ which meet at M, $BF = FM$.
Prove:
  a. $AB = MC$
  b. The medians AD and BE are perpendicular to one another.



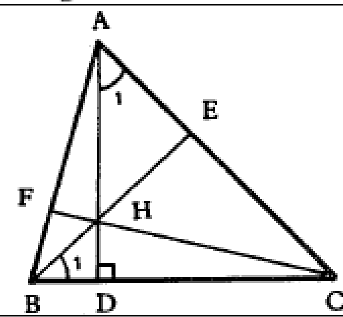
14. Given: BD is a median in $\triangle ABC$, M is the point of intersection of the medians, $BF = FM$.
Prove: $BE = \frac{BC}{5}$
Guidance: Draw from M and D segments which are parallel to AE.



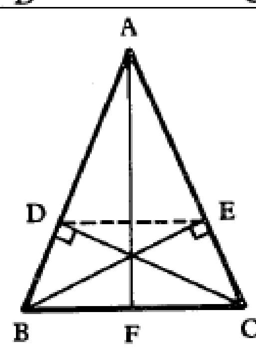
15. Given: AN, BM and CK are the three altitudes in $\triangle ABC$ and meet at point H.
Prove: Point B is the point of intersection of the altitudes of $\triangle AHC$



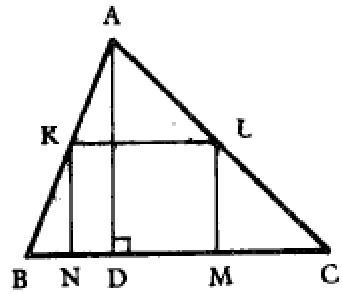
16. Given: AD is the altitude of $\triangle ABC$, $\measuredangle A_1 = \measuredangle B_1$
Prove:
  a. FC is an altitude in $\triangle ABC$
  b. $\measuredangle ABC = \measuredangle DHC$



17. Given: DC and BE are altitudes in $\triangle ABC$, $BF = FC$
Prove:
  a. $\triangle ABC$ is isosceles
  b. $BC \parallel DE$



18. Given: AD is an altitude in $\triangle ABC$, the points K, L, M and N are the midpoints of the segments AB, AC, DC and BD, respectively.
Prove:
   a. KLMN is a rectangle.
   b. Calculate the sides of the rectangle if it is given $BC = 14cm$, $AD = 10cm$



(Answer:

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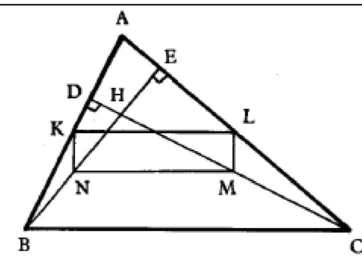
19. Given: DC and BE are altitudes in $\triangle ABC$, points K, L, M and N are the midpoints of the segments AB, AC, HC and HB, respectively.
Prove: KLMN is a rectangle.
Guidance: Draw a segment from A to BC through H.



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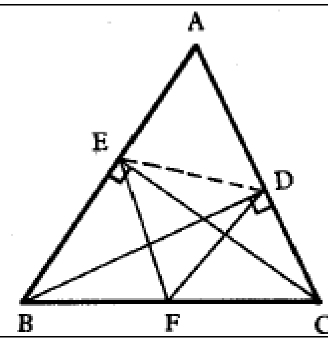
20. Given: BD and CE are altitudes in $\triangle ABC$, F is the midpoint of BC.
   a. Prove: $\triangle EFD$ is isosceles.
      Note: $\triangle ABC$ is not isosceles.
   b. Prove: $\measuredangle ABC + \measuredangle EDC = 180°$



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21. a. Given: BD and CE are altitudes in $\triangle ABC$, $ED \parallel BC$
      Prove: $\triangle ABC$ is isosceles.
      Guidance: Connect D and E with the midpoint of BC, and use the previous question.
   b. Prove the claim of section a for the case that BD and CD are angle bisectors.
   c. Is the claim of section a also valid if BD and CD are medians? Explain.

Answers:

1: a: 16 b: 7 **2**: $300cm^2$ **3**: $\frac{\sqrt{3}}{4}a^2$ **4**: a: 3 b: 10 c: 12 d: 5 **5**: a: $\frac{\sqrt{3}}{3}a$ b: $\frac{\sqrt{3}}{6}a$ **6**: $BC = 2\sqrt{2hR - h^2}$ $AB = \sqrt{2hR}$ **7**: $r = \frac{h^2 - b^2}{2h}$ $BC = \frac{h^2 - b^2}{b}$ **8**: a: 3cm b: 1cm c: $DC = \frac{\sqrt{2}}{2}a$, $r = a - \frac{\sqrt{2}}{2}a$ **9**: b: $\frac{R}{2}$, $\frac{R}{3}$ **10**: b: $\frac{R}{4}$ **18**: b: 7cm and 5 cm