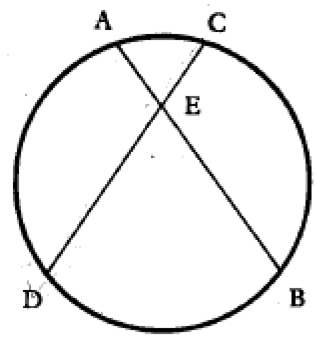
1. Given: AB and CD are two equal chords which meet at point E.
   Prove: ED = EB

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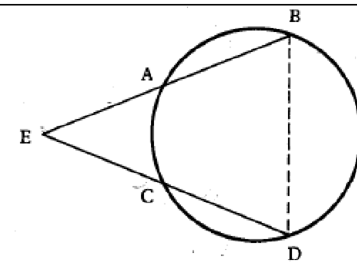
2. Given: AB and CD are two equal chords which meet at point E, external to the circle.
   Prove:
   a. Prove: EA = EC
   b. Given: $\measuredangle E = 40°$, $\overset{\frown}{BD} = 140°$
      How many degrees is $\overset{\frown}{AC}$ ?

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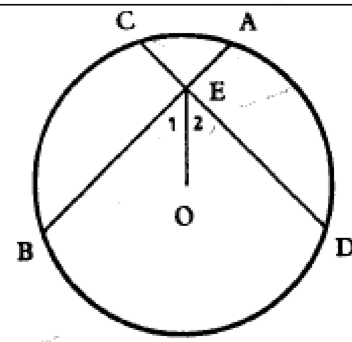
3. Given: segment OE, which connects the center of the circle to the point E, bisects the angle between the chords AB and CD ( $\measuredangle E_1 = \measuredangle E_2$ )
   Prove: AB = CD

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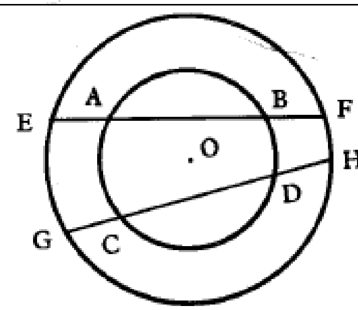
4. Given: the two circles in the drawing share a common center – 0, EF = GH
   Prove: AB = CD

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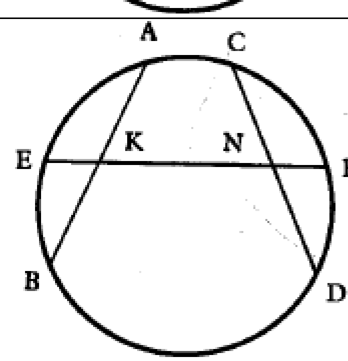
5. Given: Chord EF bisects chords AB and CD at points K and N, respectively ( AK = KB , CN = ND ),
   EK = NF
   Prove: AB = CD



6. In the following pictures, the fraction of the perimeter that $\overset{\frown}{AC}$ comprises is written on the right, and the fraction of the perimeter that $\overset{\frown}{AB}$ comprises is written on the left. Calculate the angles of $\triangle ABC$.

a: b:

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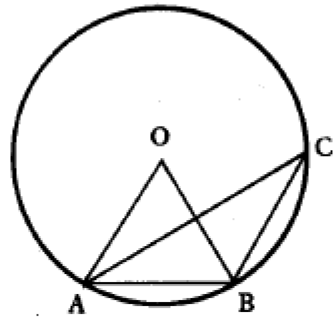
7. Given: O is the center of the circle, $\triangle ABO$ is equilateral, AC bisects $\measuredangle A$
   a. Calculate the angles of $\triangle ABC$
   b. Prove: $AO \parallel BC$

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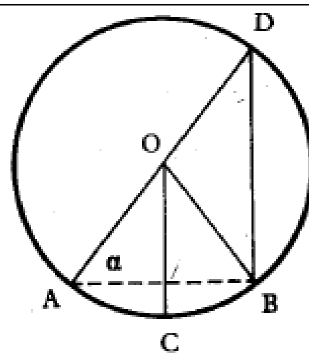
8. Given: AD is a diameter of the circle whose center is O, OC bisects $\measuredangle AOB$
   a. Prove: $BD \parallel OC$
   b. Given: $\measuredangle OAB = \alpha$
      Express $\measuredangle ODB$ in terms of $\alpha$.

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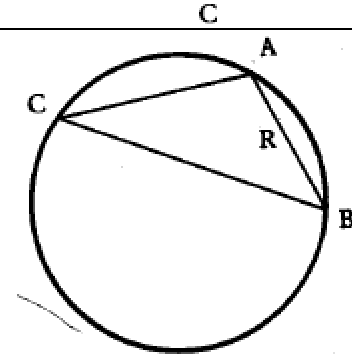
9. Given: chord AB is equal in length to the radius of the circle.
   Find $\measuredangle C$

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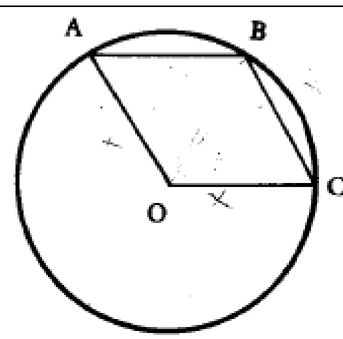
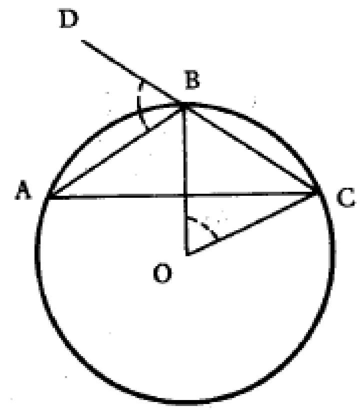
10. Given: quadrilateral ABCO is a parallelogram, such that three of its vertices are on the circle, and the fourth is the center of the circle – O
    Prove that the parallelogram is a rhombus, and find its angles.



11. Given: O is the center of the circle, AB = BC
Prove:
a. ∡BOC = ∡ABD
b. ∡ABO = ∡BCO

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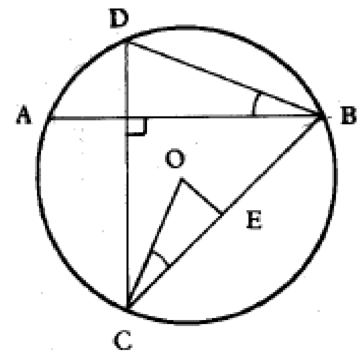
12. Given: AB and CD are two chords which are perpendicular to one another in the circle whose center is O, E is the midpoint of BC
Prove: ∡ABD = ∡OCE

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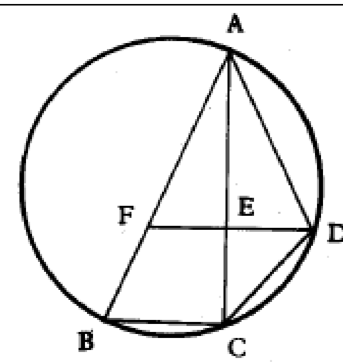
13. Given: AB is a circumference, FD is parallel to BC, FE = ED
Prove:
a. AF = AD
b. BC = CD

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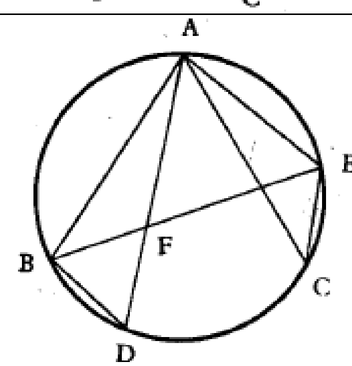
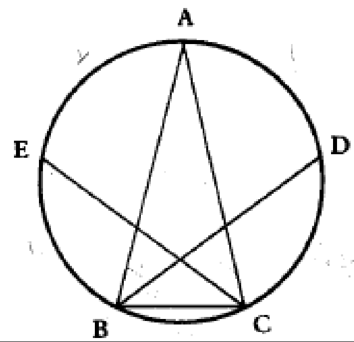
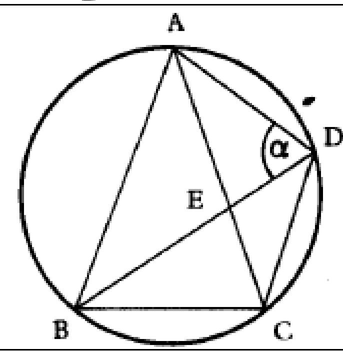
14. Given: AB = AC , CE = BF
Prove:
a. △ABF ≅ △ACE
b. BF = BD



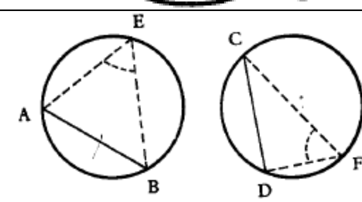
15. Given: $\triangle ABC$ is isosceles ( $AB = AC$ ), BD and EC bisect $\measuredangle B$ and $\measuredangle C$, respectively.
Prove:
a. $\widehat{BE} = \widehat{EA} = \widehat{AD} = \widehat{DC}$
b. Given: $\widehat{BC} = \frac{1}{2}\widehat{DC}$
Calculate the angles of $\triangle ABC$



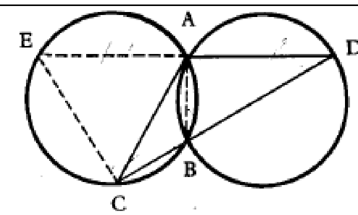

16. Given: $AB = AC$, $\widehat{AD} = \widehat{DC}$, $\measuredangle BDA = \alpha$
Express, in terms of $\alpha$: $\measuredangle BDC$ and $\measuredangle DEC$



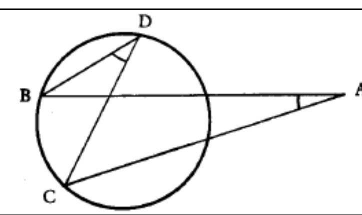

17. Given: the two circles in the drawing have equal radii, AB and CD are chords
a. Given: $AB = DC$
Prove: $\measuredangle AEB = \measuredangle CFD$
b. Prove the opposite direction of the claim in section a.



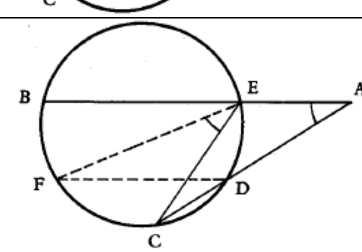

18. Given: the two circles in the drawing have equal radii, and intersect each other at points A and B, through the point B there is a segment CD.
a. Prove: $AC = AD$
Guidance: Use the previous exercise
b. Given: CD passes through the center of the circle on the right (EA is the continuation of AD)
Prove: $EA = AD$
c. Prove: $\measuredangle ECD = 90°$



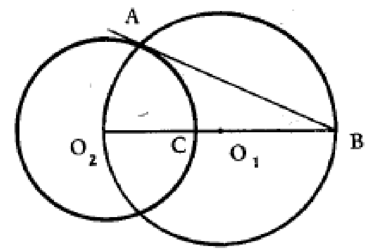

19. Given: $\measuredangle A$ is external to the circle, on $\widehat{BC}$ (on the side of the circle closer to A), there is a point D.
Prove: $\measuredangle A < \measuredangle BDC$




20. Angle $\measuredangle A$ is an external angle to the circle, $FD \parallel AB$
Prove: $\measuredangle FEC = \measuredangle A$



21. Given: two circles with centers $O_1$ and $O_2$, point A is one of the intersection points of the two circles, segment $O_2B$ is a diameter in the larger circle.
   a. Prove: AB is tangent to the small circle, at point A.
   b. Given: $\overset{\frown}{O_2A}$ (which is part of the large circle) is $50°$. Calculate how many degrees $\overset{\frown}{AC}$ (which is part of the smaller circle) is.

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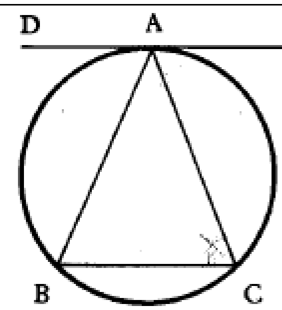
22. Given: AD is tangent to the circle at point A, AD || BC
Prove: AB = AC

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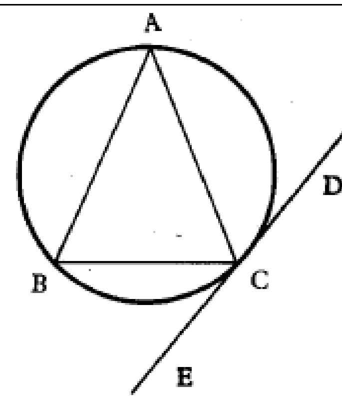
23. Given: DE is tangent to the circle at point C, $\triangle ABC$ is isosceles ( AB = AC )
   a. Prove: AC bisects $\measuredangle BCD$
   b. Given: $\measuredangle A = \alpha$
   Express, in terms of $\alpha$: $\measuredangle BCE$ and $\measuredangle ACD$

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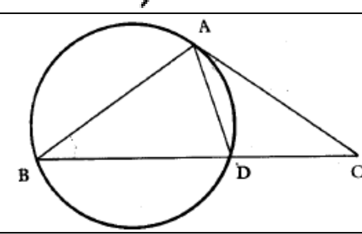
24. Given: AC is tangent to the circle at A, AB = AC
Prove: $\triangle ADC$ is isosceles.

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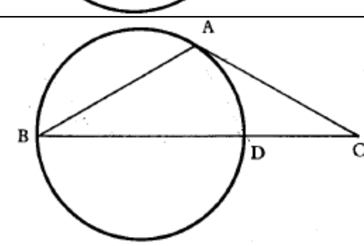
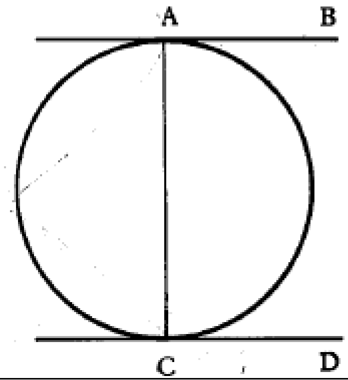
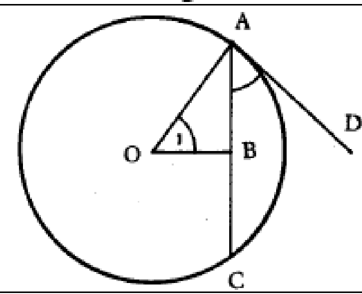
25. Given: AC is tangent to the circle at point A, $\measuredangle B = \measuredangle C = 30°$
   a. Prove: BC passes through the center of the circle.
   Guidance: connect A and D
   b. Prove: BC is three times larger than the radius of the circle.



26. Given: AB and CD are tangent to the circle at points A and C and are parallel to each other.
Prove: AC is a diameter in the circle
Guidance: Choose a point E on the circle, and connect AE and CE.

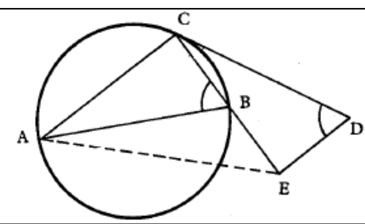



27. Given: AD is tangent to the circle, whose center is O, at point A, point B is the midpoint of chord AC.
Prove: $\measuredangle BAD = \measuredangle O_1$

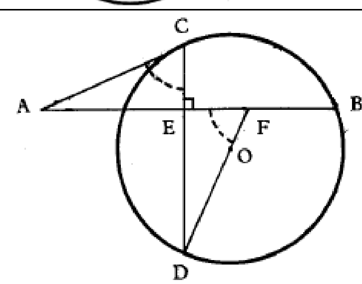



28. Given: AB is a chord in the circle whose center is O. OC is a radius which is perpendicular to chord AB. From point B there is a line tangent to the circle which intersects with the continuation of OC at point D.
Prove: BC bisects $\measuredangle ABD$

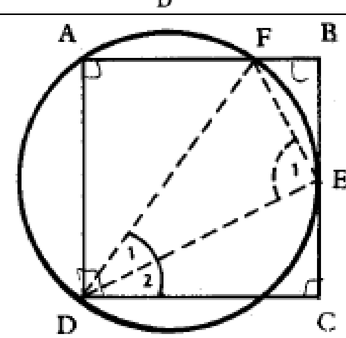
29. Given: AB is a diameter, CD is tangent to the circle at point C, $CD = AB$, $\measuredangle ABC = \measuredangle D$
   a. Prove: $\triangle ACE$ is isosceles
   b. Calculate $\measuredangle AED$



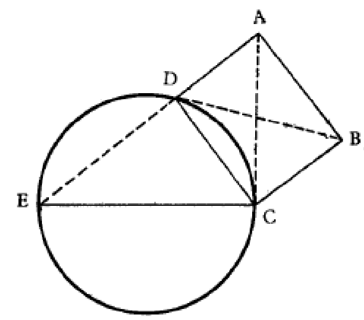

30. Given: AC is tangent to the circle, whose center is O, at point C, AB is perpendicular to chord CD.
   a. Prove: $\measuredangle C = \measuredangle F$
   b. Given: $AC = DF$
      Prove: $AF = CD$




31. Given: ABCD is a square, side BC is tangent to the circle at point E.
Prove:
   a. $\measuredangle E_1 = 90°$
   b. $\measuredangle D_1 = \measuredangle D_2$



32. Given: quadrilateral ABCD is a rectangle, EC is a diameter in the circle, diagonal BD is tangent to the circle at point D,
   a. The points D and E were connected.
      Prove: Points A, D and E line on the same line.
   b. Prove: Diagonal AC is tangent to the circle at point C.

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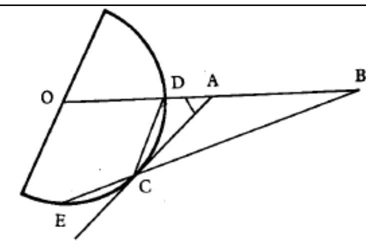
33. Given: BE intersects with the semicircle at points C and E, AC is tangent at point C and bisects $\measuredangle DCB$, the continuation of AB passes through the center – O
   a. Prove: $EC = CD$
   b. Given: $\measuredangle OAC = 40°$
      Calculate the angles of $\triangle ABC$

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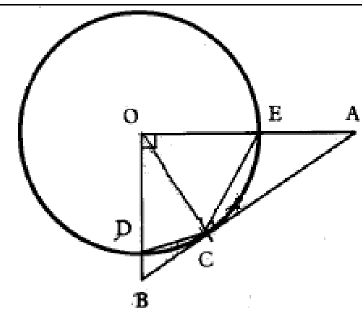
34. Given: AB is tangent to the circle at point C, $\measuredangle O = 90°$ (O is the center of the circle)
   Prove: $\measuredangle DCB + \measuredangle ECA = 45°$

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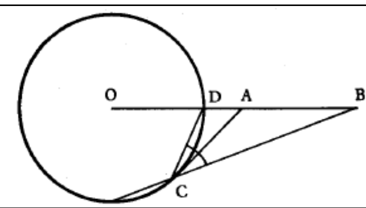
35. Given: AC is tangent to the circle, whose center is O, at C, $AB = AC$
   Prove: $\measuredangle DCB = 45°$

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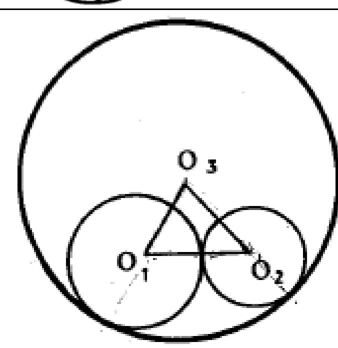
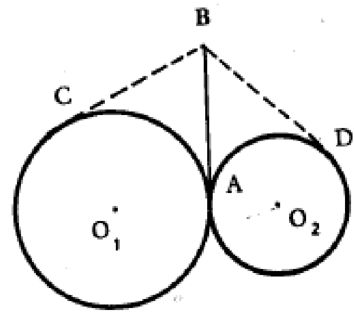
36. Given: Two circles, whose centers are $O_1$ and $O_2$, are externally tangent to each other, and internally tangent to the large circle, whose center is $O_3$.
   Prove: The perimeter of $\triangle O_1O_2O_3$ is dependant only on the radius of the large circle, and not dependant on the radius of the two small circles.



37. Given: two circles, whose centers are $O_1$ and $O_2$, touch each other at point A.

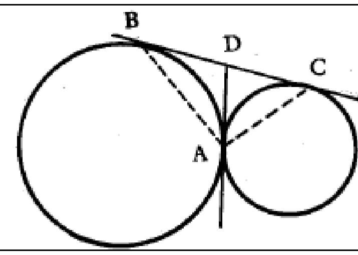
   a. Given: AB is tangent to the circle whose center is $O_1$ at point A.
   Prove: AB is also tangent at point A to the circle whose center is $O_2$.

   b. Given: BC is tangent to circle $O_1$, BD is tangent to circle $O_2$.
   Prove: $BC = BD$

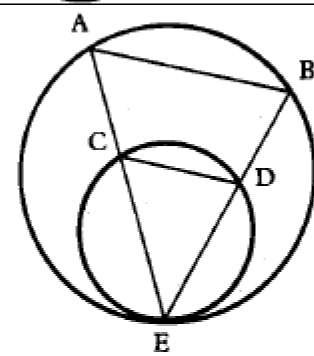



38. Given: the two circles touch each other at point A, segment BC is tangent at point B to one circle, and at point C to the other, AD is a common tangent.

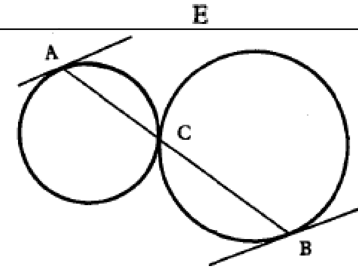
   a. D is the middle of BC.
   b. $\measuredangle BAC = 90°$



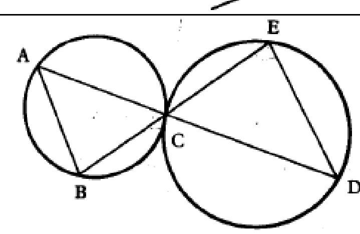

39. Given: the two circles are tangent to each other at point E.
Prove: $AB \parallel CD$
Guidance: draw a common tangent through E.



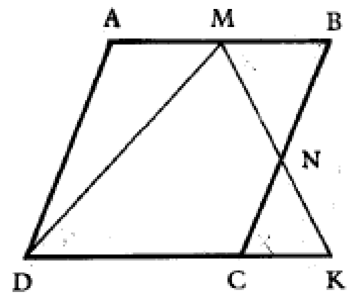

40. Given: the two circles touch each other at point C, segment AB passes through C, through A there is a tangent to the small circle, and through B there is a tangent to the large circle.
Prove: The two tangents are parallel to each other.



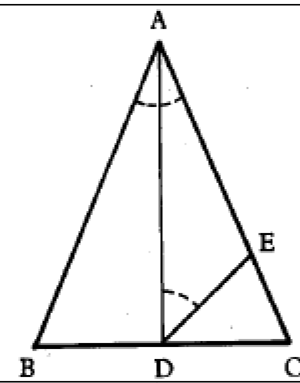

41. Given: the two circles touch each other at point C, the segments AD and BE intersect at point C.
Prove: $ED \parallel AB$



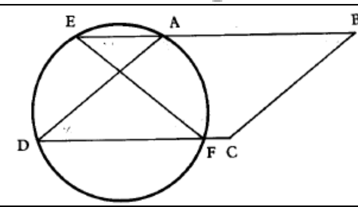
42. Given: quadrilateral ABCD is a parallelogram, $MN = BN$, $DM = DK$
Prove: It is possible to circumscribe quadrilateral MNCD with a circle.



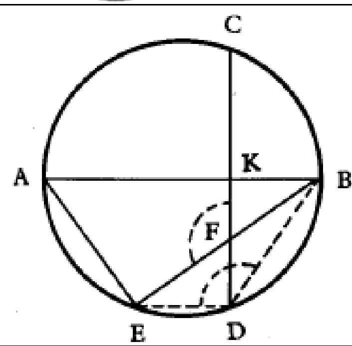

43. Given: $\triangle ABC$ is isosceles ( $AB = AC$ ), AD is a median to the base BC, $DE = DC$
   a. Prove: it is possible to circumscribe quadrilateral AEDB with a circle, and AB is the diameter of that circle.
   b. Prove: $\measuredangle A + \measuredangle ADE = 90°$



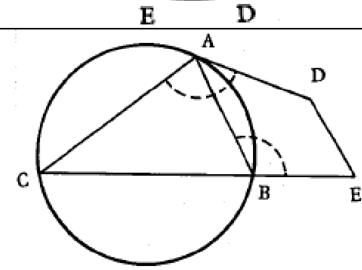

44. Given: quadrilateral ABCD is a parallelogram.
Prove: it is possible to circumscribe quadrilateral EBCF with a circle.



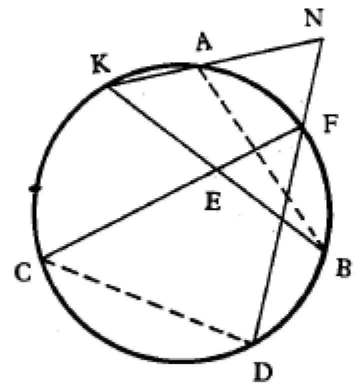

45. Given: AB is a diameter which bisects chord CD at point K, E is a point on $\overset{\frown}{AD}$
Prove:
   a. It is possible to circumscribe quadrilateral AKFE with a circle.
   b. $\measuredangle EFK = \measuredangle EDB$



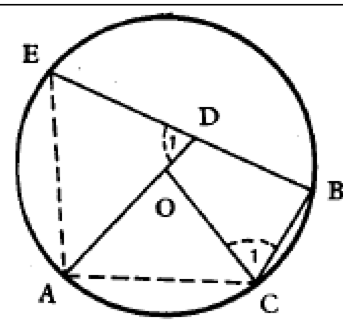

46. Given: $AB \parallel DE$, AD is tangent to the circle at point A.
Prove:
   a. It is possible to circumscribe quadrilateral CADE with a circle.
   b. $\measuredangle CAD = \measuredangle ABE$



47. Given: chords AB and CD are equal
Prove: It is possible to circumscribe quadrilateral KNFE with a circle.




48. Given: segment AD passes through the center of the circle O, AE = AC
Prove:
a. It is possible to circumscribe quadrilateral DBCO with a circle.
b. $\measuredangle C_1 = \measuredangle D_1$

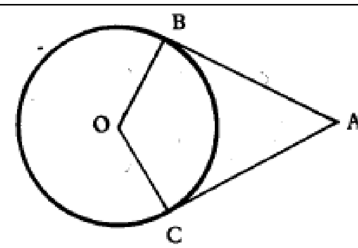



49. a. Prove: If it is possible to inscribe a circle inside an isosceles trapezoid, then the mid-segment of the trapezoid is equal to the side of the trapezoid.
b. Prove: It is not possible to inscribe a circle in the following quadrilaterals:
1. An isosceles trapezoid in which the diagonal is an angle bisector
2. An isosceles trapezoid in which the diagonals are perpendicular to each other.

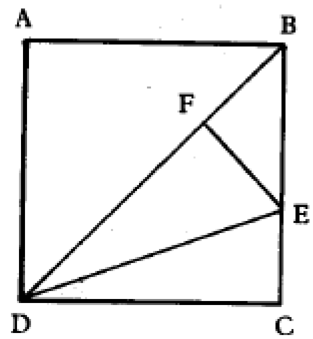
50. Which of the following shapes can be circumscribed with a circle?

a: square    b: rectangle    c: parallelogram
d: rhombus    e: trapezoid    f: kite

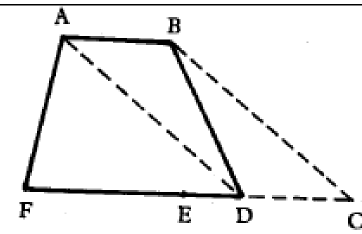
51. Given: AB and AC emanate from A and are tangent to the circle whose center is O.
Prove:
a. It is possible to inscribe a circle in quadrilateral BACO.
b. It is possible to circumscribe quadrilateral BACO with a circle.
c. The mid-point of $\overset{\frown}{BC}$ is the center of the circle which is inscribed in $\triangle ABC$



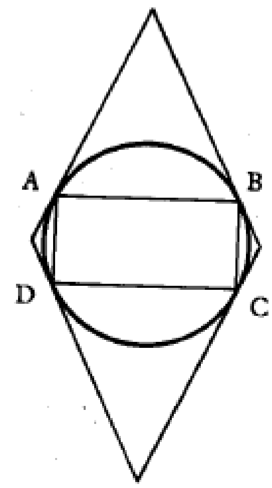
52. Given: ABCD is a square, DE bisects $\measuredangle BDC$, $EF = BF$
Prove:
    a. It is possible to circumscribe quadrilateral DEFC with a circle.
    b. It is possible to inscribe a circle in quadrilateral DEFC.



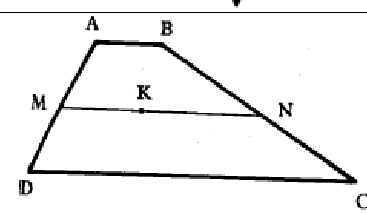

53. Given: quadrilateral ABCD is a parallelogram, E is a point on FC, $AF = EF$, $BD = EC$
Prove: It is possible to inscribe a circle in quadrilateral ABDF.




54. Given: quadrilateral ABCD is a rectangle which is inscribed in a circle.
Prove: the quadrilateral whose sides are tangent to the circle at points A, B, C and D is a rhombus.




55. Given: quadrilateral ABCD is a trapezoid, K is a point on the mid-segment MN, $MD = MK$, $NC = KN$
Prove:
    a. It is possible to inscribe a circle in trapezoid ABCD.
    b. Point K is the center of that circle.




**Answers**:
**6**: a: $36°$, $72°$, $72°$ b: $40°$, $60°$, $80°$ **7**: a: $30°$, $30°$, $120°$ **8**: b: $90-\alpha$ **9**: $30°$ **10**: $60°$ and $120°$ **15**: b: $80°$, $80°$, $20°$ **16**: $\measuredangle BDC = 180 - 2\alpha$, $\measuredangle DEC = 1.5\alpha$ **21**: b: $65°$ **23**: b: $\measuredangle BCE = \alpha$, $\measuredangle ACD = 90 - \frac{\alpha}{2}$ **29**: b: $135°$ **33**: $25°$, $15°$, $140°$