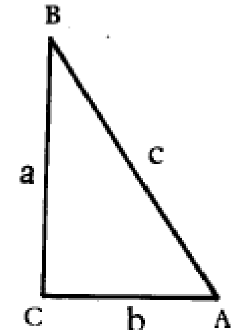
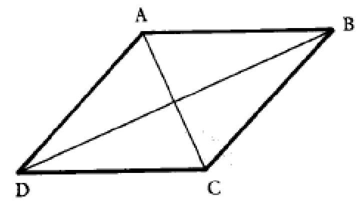
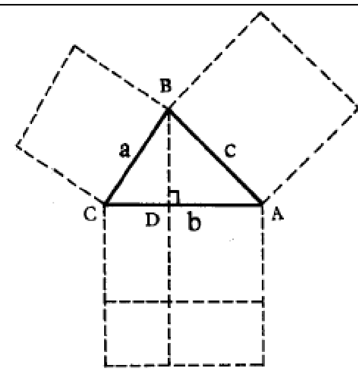
1. Prove the theorem: if the sum of the squares of two sides of a triangle equals the sum of the third side, then the triangle is a right triangle.
   Given: $a^2 + b^2 = c^2$
   Prove: $\measuredangle C = 90°$
   Guidance: build a right triangle whose sides are a and b (according to the Pythagorean theorem, the hypotenuse is equal to c, and therefore the triangles are congruent).



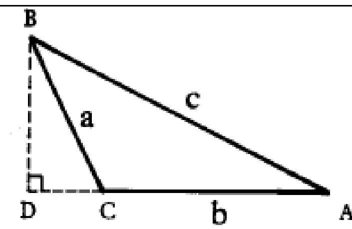
2. Given: Quadrilateral ABCD is a parallelogram, $DB = 24\text{cm}$, $AC = 10\text{cm}$, $BC = 13\text{cm}$.
   Prove: ABCD is a rhombus.



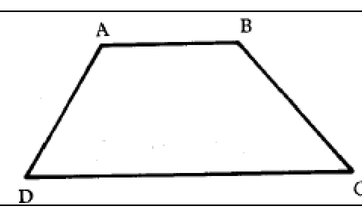
3. Prove the theorem: the area of the square built on an edge of a triangle which is opposite an acute angle is equal to the sum of the squares built on the other two sides, minus twice the area of the rectangle whose length is equal to one of the two other sides and whose width is equal to the length of the projection of the second side onto the first side.
   Given: $\measuredangle C < 90°$, CD is the projection of a on b
   Prove: $c^2 = a^2 + b^2 - 2b \cdot CD$
   Guidance: Express BD using the Pythagorean theorem, once in $\triangle ABD$ and once in $\triangle CBD$. (Equate the results and isolate c^2.)



4. Prove the theorem: the area of the square built on an edge of a triangle which is opposite an obtuse angle is equal to the sum of the squares built on the other two sides, plus twice the area of the rectangle whose length is equal to one of the two other sides and whose width is equal to the length of the projection of the second side onto the first side.
   Given: $\measuredangle C > 90°$
   Prove: $c^2 = a^2 + b^2 + 2b \cdot CD$
   Guidance: Use a similar method to the previous problem.



5. Given: ABCD is a trapezoid, $AB = 10\text{cm}$, $DC = 24\text{cm}$, $AD = 13\text{cm}$ and $BC = 15\text{cm}$.
   Calculate the area of the trapezoid.



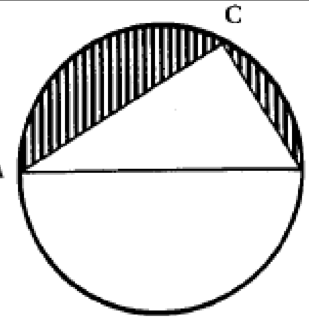
6. Find which of the following triangles has an obtuse or right angle, or which have only acute angles. The side lengths are given:
   a. 3, 7, 5 b. 2, 3, 4 c. 12, 5, 13 d. 5, 4, 6

7. In this question, the lengths of the diagonals of different parallelograms are given. The underlined number is the length of one of the sides of the parallelogram. Determine whether the given side is the longer or the shorter side of the parallelogram. Explain.
   a. 7, 8, 10   b. 9, 14, 12   c. 10, 16, 12   d. 13, 20, 16

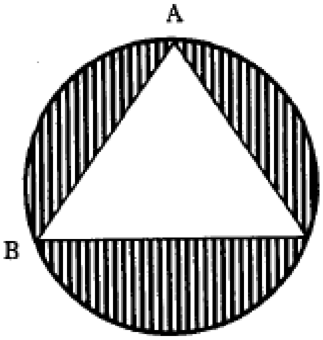
8. In this question, the lengths of two sides of different isosceles triangles are given. Determine which of the values can be the length of the base, and whether the head angle is acute or obtuse (find all of the possibilities).
   a. 10, 7   b. 5, 6   c. 7, 15   d. 6, 9   e. 5, 10

9. Calculate the perimeter and the areas of the shaded areas in the following drawings. Express the answer in terms of R and π. (In section 4, there are two tangents).

10. Given: AB is a diameter of the circle, $AC = 8cm$, $BC = 6cm$.
    Calculate the perimeter and the area of the two shaded areas together. Express the answer in terms of π.



11. Given: $\triangle ABC$, which is circumscribed by the triangle, is isosceles ($AB = AC$), $AC = 40cm$, $BC = 48cm$.
    a. Calculate the radius of the circle.
    b. Calculate the perimeter (including the inside) and the area of the shaded area.



Answers:
5: $204cm^2$ **6**: a: obtuse b: obtuse c: right d: acute **7**: a: longer b: smaller c: rhombus d: longer **8**: a: the base is 7 and the head angle is acute or the base is 10 and the head angle is obtuse b: 5 acute or 6 obtuse c: 7 acute d: 6 acute or 9 obtuse e: 5 acute **9**: a: $P = \frac{\pi R}{2} + \sqrt{2}R$, $S = \frac{\pi R^2}{4} - \frac{R^2}{2}$ b: $P = \frac{2\pi R}{3} + \sqrt{3}R$, $S = \frac{\pi R^2}{3} - \frac{\sqrt{3}R^2}{4}$ c: $P = \frac{\pi R}{3} + R$, $S = \frac{\pi R^2}{6} - \frac{\sqrt{3}R^2}{4}$ d: $P = \frac{2\pi R}{3} + 2\sqrt{3}R$, $S = \sqrt{3}R^2 - \frac{\pi R^2}{3}$ e: $P = \frac{\pi R}{3} + \sqrt{3}R + R$, $S = \frac{\pi R^2}{6}$ f: $P = \frac{\pi R}{6} + \sqrt{3}R + \sqrt{2}R$, $S = \frac{\pi R^2}{12} - \frac{\sqrt{3}R^2}{4} + \frac{R^2}{2}$ **10**: $P = 5\pi + 14cm$, $S = 12.5\pi - 24cm^2$
11: a: $25cm$ b: $P = 50\pi + 128cm$, $S = 625\pi - 768cm^2$